Problems: Tue 6/27

1. Sketch the graph of \( f(x) = 1 + 2 \cos x \).

2. Sketch the graph of \( g(x) = \frac{3x + 1}{x} \).
   
   Hint: \( \frac{a}{c} + \frac{b}{c} = \frac{a + b}{c} \).

3. Sketch the graph of the following functions.
   
   (a) \( F(x) = |\sin x| \).
   
   (b) \( G(x) = \sin(|x|) \).

4. Find the domain of \( h(x) = \frac{\tan x}{2^x \log_3(x)} \).

Even & Odd Functions

**Def:** Let \( f(x) \) be a function.

- We say \( f \) is **even** if: \( f(-x) = f(x) \). (Symmetry in \( y \)-axis)
- We say \( f \) is **odd** if: \( f(-x) = -f(x) \). (Symmetry in the origin)

5. Determine whether the following polynomials are even, odd, or neither.
   
   (a) \( p(x) = x^5 + 2x^3 + 7x \)
   
   (b) \( q(x) = x^4 - x \)
   
   (c) \( r(x) = x^6 - 3x^2 + 1 \)

   Do you see a pattern? How can you quickly tell whether a polynomial is even, odd, or neither?

6. Are there any functions that are **both** even and odd? If so, which ones?

7. If \( f \) and \( g \) are even functions, is \( f + g \) also even?
Exercises: Discontinuities: Thu 6/29

1A. Sketch the graph of \( f(x) = \begin{cases} x & \text{if } x \neq 0 \\ 2 & \text{if } x = 0. \end{cases} \)

1B. Sketch the graph of \( g(x) = \frac{x^2}{x}. \)

2A. Sketch the graph of \( \text{sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0. \end{cases} \)

2B. Sketch the graph of \( F(x) = \frac{|x|}{x}. \)

3A. Sketch the graph of \( H(x) = \begin{cases} \frac{1}{x^2} & \text{if } x < 0 \\ \sin(x) & \text{if } x \geq 0. \end{cases} \)

3B. Sketch the graph of \( K(x) = \sin\left(\frac{1}{x}\right). \)

Exercises: Continuity

4. (a) Show that \( f(x) = 2^x(x^3 - 5) \) is continuous on \((-\infty, \infty).\)

(b) Show that \( g(x) = \frac{e^x}{\sin x} \) is continuous at \( x = \frac{\pi}{2}.\)

(c) Show that \( h(x) = \cos(\ln x) \) is continuous on \((0, \infty).\)

5. Prove that the equation \( 2x + e^x = 3 \) has a solution in the interval \((0, 1).\)
1. Let \( f(x) = \frac{x - 2}{x^2 - 2x} \).

(a) Sketch the graph of \( f(x) \).

(b) Find \( f(2) \), if it exists.

(c) Find \( \lim_{x \to 2} f(x) \), if it exists.

(d) Is \( f(x) \) continuous at \( x = 2 \)? Give complete justification.

(e) Is \( f(x) \) continuous at \( x = 0 \)? Give complete justification.

(f) Is \( f(x) \) continuous at \( x = 3 \)? Give complete justification.
Problems: Wed 7/5

1. Evaluate \( \lim_{x \to 2\pi} \frac{x^3}{\cos x} \)

2. Evaluate \( \lim_{h \to 0} \frac{(3 + h)^2 - 9}{h} \)

3. Evaluate \( \lim_{t \to 0} \frac{\sqrt{t^2 + 9} - 3}{t} \)

4. Show that \( f(x) = \begin{cases} x^2 + 2 & \text{if } x > 0 \\ 2 - x & \text{if } x \leq 0 \end{cases} \) is continuous at \( x = 0 \).

5. Evaluate \( \lim_{x \to 0} \frac{|x|}{x} \)

6. Evaluate \( \lim_{x \to 0} x^8 \arctan(x) \).

7. Evaluate \( \lim_{x \to 0} x^8 \arctan\left(\frac{1}{x}\right) \).
Problems: Thu 7/6

1. Show that \( f(x) = \frac{e^x}{\sin x} \) is continuous at \( x = \frac{\pi}{2} \).

2. Show that \( g(x) = 2^x(x^3 - 5) \) is continuous on \(( -\infty, \infty )\).

3. Show that \( F(x) = \begin{cases} \frac{\sin(\pi x)}{2 - x} & \text{if } x < 1 \\ 0 & \text{if } x = 1 \\ \ln(x^2) & \text{if } x > 1 \end{cases} \) is continuous on \(( -\infty, \infty )\).

4. Prove that the equation \( 2x + 3^x = 4 \) has a solution in the interval \(( 0, 1 )\).
Problems: Vertical Asymptotes: Mon 7/10

1. Evaluate \( \lim_{x \to 1^-} \frac{x - 2}{(x - 1)^2} \) and \( \lim_{x \to 1^+} \frac{x - 2}{(x - 1)^2} \).

2. Evaluate \( \lim_{x \to 3^+} \ln(x^2 - 9) \).

3. Evaluate \( \lim_{x \to 2\pi^-} x \csc x \) and \( \lim_{x \to 2\pi^+} x \csc x \).

4. Find all vertical asymptotes of \( h(x) = \frac{x^3 - x}{x^2 - 6x + 5} \).

Problems: Horizontal Asymptotes

5. Evaluate \( \lim_{x \to \infty} \frac{x + 2}{\sqrt{9x^2 + 1}} \)

6. Evaluate \( \lim_{x \to -\infty} \frac{x + 2}{\sqrt{9x^2 + 1}} \)

7. Evaluate \( \lim_{x \to \infty} \frac{\sin^2 x}{x^3} \)
Problems: Tue 7/11

1. Evaluate \( \lim_{x \to \frac{\pi}{2}^-} e^{\tan x} \) and \( \lim_{x \to \frac{\pi}{2}^+} e^{\tan x} \).

2. Find all vertical and horizontal asymptotes of \( h(x) = e^{\frac{3}{x-2}} \).

3. Evaluate \( \lim_{x \to \infty} \left( \sqrt{x^2 + 1} - x \right) \).

4. Evaluate \( \lim_{x \to 0} x^4 e^{\left|\cos\left(\frac{1}{x}\right)\right|} \).

5. Let \( f(x) = \begin{cases} x \sin(1/x) & \text{if } x = 0 \\ 1 & \text{if } x \neq 0. \end{cases} \)

Is \( f(x) \) continuous or discontinuous at \( x = 0 \)? Fully justify your answer.

6. Evaluate \( \lim_{x \to \infty} [\ln(\sin x) - \ln(x)] \).