PREPARING MIDTERM

The second part of the quarter is on non-linear autonomous system

$$x' = F(x).$$

It roughly divides into four parts. Part 1 is on existence, uniqueness, the smooth dependence on initial condition, the maximal existence region. Part 2 is on the study of equilibrium points and its local behavior. Part 3 is on using Lyapunov functions. Part 4 is on the ω-limit of solutions.

0.1. **Existence, uniqueness and dependence on initial conditions.** For the treatment of this part, I follow closely the textbook [B]. All materials in Subsections 3.1 to 3.4 are covered; 3.5 is stated with sketchy argument. (I did not cover 3.6.)

0.2. **Equilibium.** Introduced the notion of hyperbolic equilibriums. For two dimensional equilibriums, there are classified as sink, source, and saddle. [B, Thm 4.2] is covered. For stable manifold theorem, we covered the $\mathbb{R}^2$ case, whose proof is drawn from [H, p?].

0.3. **Lyapunov functions.** Lyapunov functions are extremely powerful in analyzing a system, when exists. For a Hamiltonian system and gradient system [B, Section 4.4], such functions do exist. (Read the gradient system yourself.) The other case is for $\mathbb{R}^2$-system, where x and y in the two components of $F(x)$ are separable.

0.4. **ω-limits.** One of the key notions is *positive invariant sets*, stated in [B, Cor 5.4]. I sketched the reason, without going to its proof. The key notion is ω-limit of a solution exists on $[0, \infty)$. I outlined the proof of its properties, in [B, Sec. 5.2]. These are important. For [B, Sec 5.3], I only stated the Poincare-Bendixson Theorem [B, p.63], and showed how to apply it.

0.5. **Examples of different types of ODEs.** I went through several types of ODEs in the class. They are Subsections 6.2 and 6.3 in [B], and Subsections 11.2,

Finally, to the question what types of problems will be in the midterm, I will say the problems will be similar to those in the first three sets of assignments.

References

[B]
[H]