Problem Set VI — Math 52

Due: Thursday, Feb. 18.

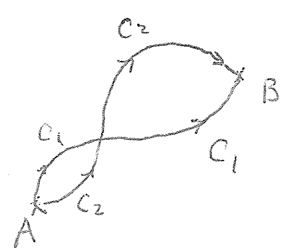
Section 14.1 (p.1085): 32, 37, 38.

Section 14.3 (p.1105): 29, 33, 34, 35, 36.

Section 14.4 (p.1114): 3, 11, 14, 17, 22, 36.

A. Let $\mathbb{F} = P\mathbf{i} + Q\mathbf{j}$ be a smooth plane vector field on the xy-plane. Suppose $\mathbb{F}$ is curl free, meaning that $curl\mathbb{F} = 0$. Apply Green's theorem to show that for two points A and B and two paths C_1 and C_2 from A to B as indicated,

$$\oint_{C_1} \mathbb{F} \cdot \mathbb{T} ds = \oint_{C_2} \mathbb{F} \cdot \mathbb{T} ds.$$



Remark:
1. Using the Green's theorem, area can be calculated by line integral along its boundary. See page 1108.
2. In the class I used the notation $\mathbb{T} = \frac{1}{v}(\dot{x}\mathbf{i} + \dot{x}\mathbf{j})$. By this I mean that after a parameterization of the curve C via $(x(t), y(t))$, along the orientation of C, then $\dot{x} = x'(t)$, the derivative in t, and v is the speed $v = \sqrt{\dot{x}^2 + \dot{y}^2}$.
3. On $dxdy$ verse dA. In the statement of Green's theorem, I have been using $dxdy$ while the textbook is using dA. If this causes any confusion (comparing with $dxdy$ or $dydx$ in iterated integrals), you can (should) follow the textbook and use dA in applying the Green's theorem.