Problem Set VI — Math 52

Due: Thursday, Feb. 18.

Section 14.3 (p.1105): 29, 33, 34, 35, 36.
Section 14.4 (p.1114): 3, 11, 14, 17, 22, 36.

A. Let \( \mathbf{F} = Pi + Qj \) be a smooth plane vector field on the \( xy \)-plane. Suppose \( \mathbf{F} \) is curl free, meaning that \( \text{curl} \mathbf{F} = 0 \). Apply Green’s theorem to show that for two points \( A \) and \( B \) and two paths \( C_1 \) and \( C_2 \) from \( A \) to \( B \) as indicated,

\[
\oint_{C_1} \mathbf{F} \cdot \mathbf{T} \, ds = \oint_{C_2} \mathbf{F} \cdot \mathbf{T} \, ds.
\]

Remark:
1. Using the Green’s theorem, area can be calculated by line integral along its boundary. See page 1108.
2. In the class I used the notation \( \mathbf{T} = \frac{1}{v}(\dot{x}i + \dot{y}j) \). By this I mean that after a parameterization of the curve \( C \) via \((x(t), y(t))\), along the orientation of \( C \), then \( \dot{x} = x'(t) \), the derivative in \( t \), and \( v \) is the speed \( v = \sqrt{\dot{x}^2 + \dot{y}^2} \).
3. On \( dxdy \) verse \( dA \). In the statement of Green’s theorem, I have been using \( dx \, dy \) while the textbook is using \( dA \). If this causes any confusion (comparing with \( dx \, dy \) or \( dy \, dx \) in iterated integrals), you can (should) follow the textbook and use \( dA \) in applying the Green’s theorem.