Math 51 Linear Algebra Review (solutions)

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Taking a Math 51 midterm in the near future? Then, this just might be for you!

Computational things to know

Parametrization

- $\underline{L = \left\{ \begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix} + t \begin{pmatrix} -2 \\ 3 \\ -7 \end{pmatrix} : t \in \mathbb{R} \right\}}$ (or others)
- $\underline{P = \left\{ \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix} + s \begin{pmatrix} -4 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ -6 \\ 0 \end{pmatrix} : s, t \in \mathbb{R} \right\}}$ (or others)

Length and angle

- $\underline{\sqrt{83}}$, $\underline{\sqrt{65}}$
- $\underline{43/\sqrt{83 \cdot 65}}$
- $\underline{\begin{bmatrix} 3 & 5 & 7 \\ 1 & 8 & 0 \end{bmatrix}}$
- $\underline{\sqrt{\|\vec{u}\|^2 + \|\vec{v}\|^2 + \|\vec{w}\|^2}}$

Column space and Nullspace

Let

$$A = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \\ j & k & l \end{bmatrix}.$$

- $\underline{2}$, <u>plane</u>, <u>are not guaranteed</u> (because a solution to $Ax = b$ exists only when b is in this plane inside $\mathbb{R}^4$; if b is not in the plane, then no solution exists)
- $\underline{1}$, <u>line</u>, <u>infinitely many and non-unique</u> (a line has infinitely many points on it!)

- $\underline{\left\{ \begin{pmatrix} a \\ d \\ g \\ j \end{pmatrix}, \begin{pmatrix} b \\ e \\ h \\ k \end{pmatrix} \right\}}$ (or others), $\underline{\left\{ \begin{pmatrix} -5 \\ 0 \\ 1 \end{pmatrix} \right\}}$ (or others)

- $\underline{\begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} + \text{span} \left\{ \begin{pmatrix} -5 \\ 0 \\ 1 \end{pmatrix} \right\}}$

- $\underline{\{\ \}}$ or $\underline{\emptyset}$ (don't exist)

- $\underline{3}$, $\underline{\text{3-space}}$, $\underline{\text{are not guaranteed}}$ (because a solution exists only if b is in this 3-dimensional subspace, which is smaller than $\mathbb{R}^4$; if b isn't, then there is no solution!)

- $\underline{0}$, $\underline{\text{point}}$, $\underline{\text{unique}}$

- $\underline{\left\{ \begin{pmatrix} a \\ d \\ g \\ j \end{pmatrix}, \begin{pmatrix} b \\ e \\ h \\ k \end{pmatrix}, \begin{pmatrix} c \\ f \\ i \\ l \end{pmatrix} \right\}}$ (or others),

- $\underline{\{\ \}}$ or $\underline{\emptyset}$

- $\underline{\left\{ \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \right\}}$

- $\underline{\left\{ \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \right\}}$

Let

$$D = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad E = \begin{bmatrix} a & b & c & d & e \\ f & g & h & i & j \\ k & l & m & n & o \end{bmatrix}.$$

- $\underline{3}$, $\underline{\text{3-space}}$, $\underline{\text{are guaranteed to}}$ (because a solution to $Ax = b$ exists whenever b is in a certain 3-dimensional subspace of $\mathbb{R}^3$; since the only such subspaces are all of $\mathbb{R}^3$, solutions must always exist)

- $\underline{2}$, $\underline{\text{plane}}$, $\underline{\text{infinitely many (and thus non-unique)}}$

- $\underline{\left\{ \begin{pmatrix} a \\ f \\ k \end{pmatrix}, \begin{pmatrix} b \\ g \\ l \end{pmatrix}, \begin{pmatrix} d \\ i \\ n \end{pmatrix} \right\}}$ (and others), $\underline{\left\{ \begin{pmatrix} -2 \\ -3 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}}$ (and others)

- $\begin{pmatrix} 3 \\ 2 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \text{span}\left\{ \begin{pmatrix} -2 \\ -3 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$ __________

- $\begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \text{span}\left\{ \begin{pmatrix} -2 \\ -3 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$ __________

Linear independence

Let

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} a & b & c & d & e \\ f & g & h & i & j \\ k & l & m & n & o \end{bmatrix}$$

be matrices such that B row reduces to A. For convenience, set

$$\vec{w} = \begin{pmatrix} a \\ f \\ k \end{pmatrix} \quad \vec{x} = \begin{pmatrix} b \\ g \\ l \end{pmatrix} \quad \vec{y} = \begin{pmatrix} c \\ h \\ m \end{pmatrix} \quad \vec{z} = \begin{pmatrix} d \\ i \\ n \end{pmatrix}.$$

- dependent, because $2\vec{w} + 3\vec{x} - \vec{y} = 0$
- independent, because the first, second and fourth columns of A are
- independent, because the second, third and fourth columns of A are
- $\{\vec{w}, \vec{x}, \vec{z}\}$, $\{\vec{x}, \vec{y}, \vec{z}\}$, $\{\vec{w}, \vec{y}, \vec{z}\}$