Math 51 Linear Algebra Review

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Taking a Math 51 midterm in the near future? Then, this just might be for you!

Computational things to know

Parametrization

- given two vectors $\begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 8 \\ 0 \end{pmatrix}$, a parametric equation for the line in $\mathbb{R}^3$ containing them is ____________________________
- given three vectors $\begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$, a parametric equation for the plane in $\mathbb{R}^3$ containing them is ____________________________________

Length and angle

- given two vectors $\begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 8 \\ 0 \end{pmatrix}$, their lengths are __________ and __________
- given two vectors $\begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 8 \\ 0 \end{pmatrix}$, the angle between them is arccos_______________
- given two vectors $\begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 8 \\ 0 \end{pmatrix}$, the set of vectors in $\mathbb{R}^3$ orthogonal to both of them is precisely the nullspace of ____________________________
- if $\vec{u}$, $\vec{v}$ and $\vec{w}$ are orthogonal vectors, then the length $\|\vec{u} + \vec{v} + \vec{w}\|$ can be rewritten as __________________________

Column space and Nullspace

Let

$$A = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \\ j & k & l \end{bmatrix}.$$

- the rank of A is __________, so the set of vectors $b \in \mathbb{R}^4$ such that $Ax = b$ has a solution forms a ________________ and thus solutions ________________ exist
- the nullity of A is __________, so for the vectors $b \in \mathbb{R}^4$ such that a solution exists, the set of solutions forms a ________________ and thus solutions are ________________
- if C is a matrix that row-reduces to A, then a basis for the column space of C is

  and a basis for the nullspace of C is

  ________________________ ________________________

- the set of solutions to $Ax = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 0 \end{pmatrix}$ is ____________________________
- the set of solutions to $Ax = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 0 \end{pmatrix}$ is ____________________________

- the rank of B is __________, so the set of vectors $b \in \mathbb{R}^4$ such that $Bx = b$ has a solution forms a ________________ and thus solutions ________________ exist
- the nullity of B is __________, so for the vectors $b \in \mathbb{R}^4$ such that a solution exists, the set of solutions forms a ________________ and thus solutions are ________________
- if C is a matrix that row-reduces to B, then a basis for the column space of C is

  and a basis for the nullspace of C is

  ________________________ ________________________

- the set of solutions to $Bx = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 0 \end{pmatrix}$ is ____________________________
- the set of solutions to $Bx = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 0 \end{pmatrix}$ is ____________________________

Let

$$D = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad E = \begin{bmatrix} a & b & c & d & e \\ f & g & h & i & j \\ k & l & m & n & o \end{bmatrix}.$$

- the rank of D is __________, so the set of vectors $b \in \mathbb{R}^3$ such that $Dx = b$ has a solution forms a _______________ and thus solutions _______________ exist
- the nullity of D is __________, so for the vectors $b \in \mathbb{R}^3$ such that a solution to $Dx = b$ exists, the set of solutions forms a _______________ and thus solutions are _______________
- if E is a matrix that row-reduces to D, then a basis for the column space of E is

  and a basis for the nullspace of E is

  ______________________ ______________________
- the set of solutions to $Dx = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ is _________________________
- the set of solutions to $Dx = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ is _________________________

Linear independence

Let

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} a & b & c & d & e \\ f & g & h & i & j \\ k & l & m & n & o \end{bmatrix}$$

be matrices such that B row reduces to A. For convenience, set

$$\vec{w} = \begin{pmatrix} a \\ f \\ k \end{pmatrix} \quad \vec{x} = \begin{pmatrix} b \\ g \\ l \end{pmatrix} \quad \vec{y} = \begin{pmatrix} c \\ h \\ m \end{pmatrix} \quad \vec{z} = \begin{pmatrix} d \\ i \\ n \end{pmatrix}.$$

- the vectors $\vec{w}, \vec{x}, \vec{y}$ are independent/dependent, because ___________________________________
- the vectors $\vec{w}, \vec{x}, \vec{z}$ are independent/dependent, because ___________________________________
- the vectors $\vec{x}, \vec{y}, \vec{z}$ are independent/dependent, because ___________________________________
- using elements from the set $\{\vec{w}, \vec{x}, \vec{y}, \vec{z}\}$, three bases of $\mathbb{R}^3$ are _______________, _______________ and _______________

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