Math 51 Linear Algebra Checklist

Jonathan Lee

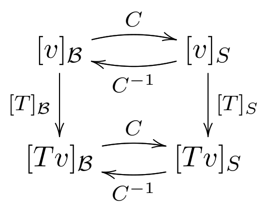
December 9, 2007

Taking a Math 51 final examination in the near future? Then, this just might be for you!

Useful concepts to know:

- $n \times m$ matrices represent functions $\mathbb{R}^m \to \mathbb{R}^n$
- composition of functions corresponds to multiplying their representing matrices
- why are nullspace, row space, column space, image space and whatever-space necessarily subspaces?
- what does it mean for dot product to be zero? for cross-product to be zero?
- what are two interpretations of matrix-vector product? why does it matter?
- every set of vectors spans some subspace, but is not necessarily a basis for that subspace — why not?
- what do rank(A) and nullity(A) say about the existence and uniqueness of solutions to the equation $Ax = b$?
- in the formula giving a matrix for the orthogonal projection onto a line, why is it necessary that the line pass through the origin? or that the vector chosen to span the line have unit length?
- can you derive the matrix representing reflection around a line just from knowing the projection matrix?

- why is a square matrix invertible if and only if it has non-zero determinant?
- what are consequences of the following diagram? (what does C have to be?) can you give formulas relating different matrices? can you say something about eigenvectors and eigenvalues?



Useful computational things to know:

- parametrizing a line
- determining the angle between two vectors
- testing a set of vectors for linear independence
- finding a linearly independent subset of a given set of vectors that spans the same subspace
- computing (perhaps even finding bases of) the nullspace and column space of a matrix
- computing the area of a parallelogram in $\mathbb{R}^2$; computing the volume of a paralleliped in $\mathbb{R}^3$
- computing a determinant efficiently; know how to choose between the row-reduction and block-sum expansion methods!
- testing a 2×2 matrix for definiteness without actually computing eigenvalues

Useful argumentations to know:

- how do you show something is a subspace without resorting to verifying each axiom?
- if you know something about the eigenvalues of A and B, know how to show stuff about those of A^{-1} and $A^{-1}BA$ (when these expressions make sense)