1, Assume that $\Omega = [0, 1]$ and $\mathcal{F} = \sigma(\{[0, 1/2], (1/2, 1]\})$. What is the maximal number of values a random variable on $(\Omega, \mathcal{F})$ can take? What if $\mathcal{F} = \sigma(\{[0, 1/3], [2/3, 1]\})$?

Exercises in Dembo’s lecture notes: Exercise 1.2.12, Exercise 1.2.22, Exercise 1.2.27, Exercise 1.2.31.

Notice: There is a typo in Exercise 1.2.12 (as pointed out by Casey Lynne McKnight). In part (a), you want to show that $\{x : g(x) \leq a\}$ is closed. Alternatively, you may show instead that $\{x : g(x) > a\}$ is an open set for each $a \in \mathbb{R}$.