If $f : \mathbb{R}^n \to \mathbb{R}$ attains a local minimum at $a$, then the restriction of $f$ to any line through $a$ attains a local minimum at $a$. In this problem, you will show that the converse is not true.

Let $f(x, y) = (y - x^2)(y - 3x^2)$.

1. Show that $f$ has a critical point at $(0, 0)$ and compute $H_f(0, 0)$.

2. Use the single variable Second Derivative Test to show that the restriction of $f$ to any line through the origin has a local minimum at $(0, 0)$.

3. Show that $(0, 0)$ is not a local minimum for $f$. 