Today’s topic:

- linear combination

**Definition (linear combination)** A *linear combination* of a set of vectors \( \{v_1, \cdots, v_k\} \) in \( \mathbb{R}^n \) is a vector of the form

\[
\mathbf{v} = c_1 v_1 + \cdots + c_k v_k
\]

**Example 1.**

1. Write the zero vector as a linear combination of \[
\begin{pmatrix} 2 \\ 1 \\ -4 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix}.
\]

2. What about the zero vector as a linear combination of \[
\begin{pmatrix} 2 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{? (zero coefficients work!)}
\]

3. Write the vector \[
\begin{pmatrix} x \\ \pi \end{pmatrix}
\] (where \( x \) is a variable, and \( \pi \) is the constant we all know) as a linear combination of \[
\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} \pi \\ 1 \end{pmatrix} \text{?}
\]

Solution. Which vectors occur as linear combinations of these two? We have

\[
\begin{pmatrix} 2 \\ 1 \end{pmatrix} - \begin{pmatrix} \pi \\ 1 \end{pmatrix} = \begin{pmatrix} 2 - \pi \\ 0 \end{pmatrix}
\]

as a linear combination, and hence also

\[
\frac{1}{2 - \pi} \left( \begin{pmatrix} 2 \\ 1 \end{pmatrix} - \begin{pmatrix} \pi \\ 1 \end{pmatrix} \right) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\]

So then we also have as a linear combination of \( v_1, v_2 \)

\[
\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\]

\[
= \cdots
\]

Finally, this is how we can write \[
\begin{pmatrix} x \\ \pi \end{pmatrix}
\] as a linear combination of \( v_1, v_2 \).