MATH 41
TA Section Notes for Tue 09 Oct 07
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Admin:
• give out Mark’s handouts on limits?
• return weekly HW 1
• collect weekly HW 2

Today:
• more on limits
• (instantaneous) rates of change

1 More on Limits

Section 2.5, Ex 21  This is analogous to Example 5, p.134.

Section 2.5, Ex 25  Since there is a radical sign, we can multiply the function by its conjugate to see what we get:

\[
\sqrt{9x^2 + x - 3x} = \left(\sqrt{9x^2 + x - 3x}\right) \cdot \frac{\sqrt{9x^2 + x + 3x}}{\sqrt{9x^2 + x + 3x}}
\]

\[
= \frac{(9x^2 + x) - (3x)^2}{\sqrt{9x^2 + x + 3x}} \cdot \frac{x}{\sqrt{9x^2 + x + 3x}}
\]

\[
= \frac{1}{\sqrt{9 + \frac{1}{2} + 3}} \quad \text{(multiply both numerator and denominator by 1/x)}
\]

Now we can evaluate the limit easily:

\[
\lim_{x \to \infty} \sqrt{9x^2 + x - 3x} = \lim_{x \to \infty} \frac{1}{\sqrt{9 + \frac{1}{2} + 3}} = \frac{1}{\sqrt{9 + 0 + 3}} = 1/6.
\]

Section 2.5, Ex 29  Use the squeeze theorem.

Note that \(-1 \leq \cos x \leq 1\) for all \(x\). Multiplying this throughout by \(e^{-2x}\) (which is always positive), we get

\[-e^{-2x} \leq e^{-2x} \cos x \leq e^{-2x}\]

Then take \(\lim_{x \to \infty}\) throughout and use the squeeze theorem.
Section 2.5, Ex 41  Here’s how you can go about figuring out such a function $f(x)$ from scratch:

What functions satisfy $\lim_{x \to 3^-} f(x) = \infty, \lim_{x \to 3^+} f(x) = -\infty$? $f(x) = \frac{1}{x-3}$ is such an example. (If you take $\frac{1}{x}$, the signs would be the opposite of what we want.)

To get $f(2) = 0$, we can modify our function to $f(x) = x - \frac{2}{3} - x$. Note that this function still satisfies $\lim_{x \to 3^-} f(x) = \infty, \lim_{x \to 3^+} f(x) = -\infty$.

How do we get $\lim_{x \to 0} f(x) = -\infty$ as well? At this stage, our function $f(x) = \frac{x-2}{3} - x$ is such that $\lim_{x \to 0} f(x) = -\frac{2}{3},$ which is negative. We know that $\lim_{x \to 0} \frac{1}{x} = \infty$, so we try $f(x) = \frac{x-2}{3} - \frac{1}{x}$, which indeed satisfies $\lim_{x \to 0} f(x) = -\infty$, as well as $f(2) = 0, \lim_{x \to 3^-} f(x) = \infty, \lim_{x \to 3^+} f(x) = -\infty$.

We note that the remaining condition $\lim_{x \to \pm \infty} f(x) = 0$ is already satisfied by our $f(x)$ as it is.

2  (Instantaneous) Rate of Change

Section 2.6, Ex 27  (b) According to the formula on p.144, if we have a function $y = f(x)$, then the instantaneous rate of change of $y$ with respect to $x$ at $x = x_1$ is given by

$$\lim_{x \to x_1} \frac{f(x) - f(x_1)}{x - x_1}.$$

Hence the instantaneous rate of change we are looking for here is

$$\lim_{x \to 100} \frac{C(x) - C(100)}{x - 100} = \lim_{x \to 100} \frac{5000 + 10x + 0.05x^2 - 6500}{x - 100}$$

$$= \lim_{x \to 100} \frac{0.05x^2 + 10x - 1500}{x - 100}$$

$$= \lim_{x \to 100} \frac{(x + 300)(x - 100)}{x - 100}$$

$$= \lim_{x \to 100} x + 300$$

$$= 400$$

Section 2.7, Ex 29  (a) The derivative is a measure of rate of change. Hence $f'(v)$ is the rate of change of fuel consumption. The units are

$$\frac{\text{units of } f(v)}{\text{units of } v} = \frac{\text{gallons/hr}}{\text{miles/hr}}$$

(b) $f'(20)$ is the rate of change of fuel consumption when the velocity is $v = 20$ (miles/hr). $f'(20) = -0.05$ is negative, so this means that when the car is travelling at 20 miles per hour, the fuel consumption will decrease if we increase the velocity by a little (do a sketch to illustrate this).