Northern California
Symplectic Geometry Seminar

Berkeley – Davis – Santa Cruz – Stanford

Monday, February 4th, at **Stanford**

2:30–3:30, room 383N, **Larry Guth (Stanford)**

"Symplectic embeddings of polydisks"

Abstract: If P and P' are two symplectic polydisks, then there are two well-known obstructions to embedding P into P': the total volume and Gromov's non-squeezing theorem. We prove that up to a constant factor these are the only obstructions.

This talk will focus on putting the result in context. We will look at it from the point of view of symplectic capacities, from the point of view of Riemannian geometry, and from the point of view of classical physics. Then I will describe the main ideas of the proof.

3:30–4:00 — Tea Break, 2nd floor lounge

4:00–5:00, room 383N, **Ronny Hadani (Chicago)**

"Quantization of symplectic vector spaces: algebra-geometric approach"

Abstract: Quantization is a fundamental procedure in mathematics and in physics. From the physical side, quantization is the procedure by which one associates to a classical mechanical system its quantum counterpart. From the mathematical side, it seems that quantization is a way to construct interesting Hilbert spaces out of symplectic manifolds, suggesting a method for constructing representations of the corresponding groups of symplectomorphisms.

The problem of quantization can be described as follows. Given a symplectic manifold (M, ω) one would like to associate to it, in a functorial manner, an Hilbert space $\mathcal{H}_M$, such that, in particular, if (M, ω) is acted upon by a suitable group G of symplectomorphisms then the Hilbert space $\mathcal{H}_M$ supports a representation

$$\rho : G \to U(\mathcal{H}_M).$$

In my lecture, I will consider the problem of quantization in the simplified setting of symplectic vector spaces over the finite field $\mathbb{F}_p$. Specifically, I will construct a quantization functor, $\mathcal{H}$, associating a Hilbert space $\mathcal{H}_V$, to a finite dimensional symplectic vector space (V, ω) over $\mathbb{F}_p$. As a result, we will obtain a canonical model for the Weil representation of the symplectic group $Sp(V)$.

The main technical result, is a proof of a strong form of the Stone-von Neumann theorem for the Heisenberg group over $\mathbb{F}_p$. This result, roughly, concerns the existence of a canonical flat connection on a certain vector bundle $\mathcal{H}$, defined on $Lag(V)$. In this terminology, the space $\mathcal{H}_V$ is obtained as the space of horizontal sections in $\mathcal{H}$.

The connection is constructed as follows: It is given explicitly as a system of isomorphisms between pairs of fibers, $F_{M,L} : \mathcal{H}_{|L} \stackrel{\simeq}{\to} \mathcal{H}_{|M}$, for $M, L \in Lag(V)$ which are in transversal position, i.e., $M \cap L = 0$. The construction of $F_{M,L}$, for general $M, L \in Lag(V)$, is obtained from the transversal formulas using the algebra-geometric operation of (perverse) extension.

Joint work with: Shamgar Gurevich (Berkeley)

Please contact ionel@math.stanford.edu to arrange parking.
There will be a dinner at 6pm.

—Y. Eliashberg
D. Fuchs
V. Ginzburg
E. Ionel
R. Montgomery
A. Weinstein