Abstract: If $L \subset X$ is an exact Lagrangian submanifold of an exact symplectic manifold with convex end $\Lambda \subset Y$, where $Y$ is a contact manifold and $\Lambda$ is a Legendrian submanifold, and if $L$ has empty concave end, then the contact homology of $\Lambda$ can be linearized with respect to $L$. Following ideas of P. Seidel, we introduce a version of Lagrangian Floer cohomology of $L$ which together with the linearized contact cohomology of $\Lambda$ and the singular homology of $L$ fit into an exact triangle. In the special case $X = \mathbb{C}^n$, the exact triangle leads to an isomorphism between the linearized Legendrian contact cohomology of $\Lambda \subset S^{2n-1}$ and the singular homology of $L$. Under this isomorphism, the duality exact sequence for linearized contact homology correspond to the homology sequence of the pair $(L, \Lambda)$. 