Problem 1. Assume $\alpha$ is a skewsym bilinear form on $V$. Show that there exists a basis $\{u_1, \ldots, u_n, v_1, \ldots, v_n, w_1, \ldots, w_k\}$ of $V$ such that $\alpha = \sum_{j=1}^{n} u_j^* \wedge v_j^*$, i.e. the corresponding matrix is
\[
\begin{pmatrix}
0 & I_n & 0 \\
-I_n & 0 & 0 \\
0 & 0 & 0_k
\end{pmatrix}
\]
Note that $k = \dim \ker \tilde{\alpha}$ and $2n = \rank \tilde{\alpha}$.

Problem 2. Show that any $2n$ dimensional symplectic vector space $(V, \omega)$ is (linearly) symplectomorphic to $(\mathbb{R}^{2n}, \omega_0)$.

Problem 3. Assume $\alpha$ is a skewsym bilinear form on $V$. Show that $\tilde{\alpha}$ is invertible iff $\alpha^\wedge n \neq 0$.

Problem 4. Prove the following properties for the symplectic complement $W^\omega$
(a) $\dim W + \dim W^\omega = \dim V$
(b) $(W^\omega)^\omega = W$
(c) $(W_1 \cap W_2)^\omega = W_1^\omega + W_2^\omega$
(d) if $W_1 \subset W_2$ then $W_2^\omega \subset W_1^\omega$

Problem 5. Prove that we can always find coordinates on $(V, \omega)$ such that a sympl, isotopic, coisotopic or respectively Lagrangian subspace $W$ looks like the standard example.

Problem 6. Show that any basis $\{x_1, \ldots, x_n\}$ of a Lagrangian subspace $W$ can be completed to a symplectic basis of $(V, \omega)$. In fact, $(V, \omega) \cong_{\text{sympl}} (W \oplus W^*, \omega_{\text{can}})$.

More generally, prove the following (equivalent) statements:

Problem 7. (local coordinates) Assume $W$ is a lin subspace of a symplectic vector space $(V, \omega)$. Show that there exists a symplectic basis $\{x_1, \ldots, x_n, y_1, \ldots, y_n\}$ of $V$ such that:
(a) $W = \text{span}\{x_1, \ldots, x_{k+l}, y_1, \ldots, y_k\}$
(b) $W^\omega = \text{span}\{x_{k+1}, \ldots, x_n, y_{k+l+1}, \ldots, y_n\}$
and therefore $W \cap W^\omega = \text{span}\{x_{k+1}, \ldots, x_{k+l}\}$.

Problem 7. (intrinsic) Assume $W$ is a linear subspace of a symplectic $(V, \omega)$, and let $N = W \cap W^\omega = \ker (\omega|_W)$. Show that:
(a) $(W/N, \omega)$ is a symplectic vector space
(b) $(V, \omega) \cong_{\text{sympl}} (W/N, \omega) \oplus (W^\omega/N, \omega) \oplus (N \oplus N^*, \omega_{\text{can}})$