1 Bucket O' Ye Olde Troo/False

1)Every linear transformation on $\mathbb{R}^n$ has distinct eigenvalues.

False. Counterexample is the identity matrix I_n which only has one eigenvalue.

2) If a real matrix has one eigenvector, then it has an infinite number of eigenvectors.

True. If $\overrightarrow{v}$ is an eigenvector, then $a\overrightarrow{v}$ is an eigenvector for any scalar $a \in \mathbb{R}$.

3) There exists a square matrix with no real eigenvectors.

True. As an example, take the rotation matrix

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

4) Eigenvalues must be non-zero scalars.

False. Zero is a perfectly good eigenvalue. (The zero matrix, or any projection matrix is an example of a matrix whose eigenvalues include zero.)

5) Any two eigenvectors are linearly independent.

False. Consider the vector $\overrightarrow{v}$ and any linear multiple of $\overrightarrow{v}$. 5) would be true if their associated eigenvalues were distinct.

6) The sum of two eigenvalues of a linear transformation T is also an eigenvalue of T.

False. There is absolutely no reason to think this is true. Again, take the identity matrix. 1 is an eigenvalue, yet 1+1=2 and 2 is not an eigenvalue.

7) Similar matrices have the same eigenvalues.

True. Because

$$\det(CAC^{-1} - \lambda I) = \det(CAC^{-1} - \lambda CC^{-1})$$
$$= \det(C(A - \lambda I)C^{-1})$$
$$= \det(C)\det(A - \lambda I)\det(C^{-1})$$
$$= \det(A - \lambda I)$$

So the characteristic polynomials of similar matrices are the same. Thus they have the same eigenvalues. Or see the second eigenstuff handout.

8) If A and B are similar matrices, λ is an eigenvalue for A with eigenspace A_λ and λ is an eigenvalue for B with eigenspace B_λ, then $\dim A_\lambda = \dim B_\lambda$.

True! See the second eigenstuff handout.

9) Similar matrices have the same eigenvectors.

False! Again, see the second eigenstuff handout.

10) The sum of two eigenvectors of a linear tranformation T is always an eigenvector.

False. This would be true if the eigenvectors had the same eigenvalue.

11) Any linear transformation on $\mathbb{R}^n$ that has fewer than n distinct eigenvalues is not diagonalizable.

False. Take the identity matrix. It is diagonal and has only eigenvalue 1.

12) Eigenvectors corresponding to the same eigenvalue are always linearly dependent.

False. Again, take the identity matrix. It is diagonal, so it has an eigenbasis (in particular linearly independent), and all eigenvectors have associated eigenvalue 1.

13) If λ is an eigenvalue of T, then each element of E_λ is an eigenvector of T.

False on technicality! $\overrightarrow{0} \in E_\lambda$ but $\overrightarrow{0}$ is not allowed to be an eigenvector.

14) If λ_1, λ_2 are distinct eigenvalues of T, then $E_{\lambda_1} \cap E_{\lambda_2} = \{\overrightarrow{0}\}$.

True. Suppose there is some $\overrightarrow{x}$ in both E_{λ_1} and $E_{\lambda 2}$. Then $T\overrightarrow{x} = \lambda_1 \overrightarrow{x}$ and $T\overrightarrow{x} = \lambda_2 \overrightarrow{x}$ which would imply that $\lambda_1 \overrightarrow{x} = \lambda_2 \overrightarrow{x}$. The only way this can hold is for $\lambda_1 = \lambda_2$ (but we supposed our eigenvalues were distinct, so this isn't the case) or for $\overrightarrow{x} = \overrightarrow{0}$.

15) Let A be a n×n matrix. Let $\beta = \{\overrightarrow{v}_1, ..., \overrightarrow{v}_n\}$ be a basis for $\mathbb{R}^n$ consisting of eigenvectors of A. If

$$Q = \begin{bmatrix} | & & | \\ \overrightarrow{v}_1 & \cdots & \overrightarrow{v}_n \\ | & & | \end{bmatrix}$$

then $Q^{-1}AQ$ is a diagonal matrix.

True. Alright, since β is an eigenbasis for A, if we write A in β coordinates, A_β, we should get a diagonal matrix. The change of basis stuff we discussed earlier tells us that $A = QA_\beta Q^{-1}$. By multiplying both sides on the left by Q^{-1} and both sides on the right by Q, we get $Q^{-1}AQ = A_\beta$ which is diagonal.

16)Let A be a n×n matrix. Let $\beta = \{\overrightarrow{v}_1, ..., \overrightarrow{v}_n\}$ be a basis for $\mathbb{R}^n$ consisting of eigenvectors of A. If

$$Q = \begin{bmatrix} | & & | \\ \overrightarrow{v}_1 & \cdots & \overrightarrow{v}_n \\ | & & | \end{bmatrix}$$

then QAQ^{-1} is a diagonal matrix.

False. The Q's are in the wrong order here.