1 A Guide On Row Reducing

This pdf aims to be a clear guide on how to row reduce consistently, without errors. To see why putting matrices in Row Reduced Echelon Form is important, please read "Row Echelon Form 9/30".

· To row reduce we use 3 kinds of "moves" to transform a matrix in row reduced echelon form.
The legal moves are:
i) Swapping rows

$$\begin{bmatrix} 1 & -3 & 3 \\ 0 & 1 & 1 \\ 2 & 0 & -1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & -3 & 3 \\ 2 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

Swap row 2 and row 3. ii) Multiplying a row by a <u>*nonzero*</u> constant

$$\begin{bmatrix} 1 & -3 & 3 & 0 \\ 0 & 1 & 1 & -1 \\ 2 & 0 & -1 & 6 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & -3 & 3 & 0 \\ 0 & 1 & 1 & -1 \\ -6 & 0 & 3 & -18 \end{bmatrix}$$

Scale row 3 by -3. iii) Adding a multiple of a row to any other row.

$$\begin{bmatrix} 2 & -1 \\ 0 & 0 \\ 4 & 7 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 2 & -1 \\ 0 & 0 \\ -2 & 10 \end{bmatrix}$$

Add -3×row 1 to row 2.

· From person to person, the order in which these moves are used may vary. Other TAs may have slightly different methods for row reducing. This is fine, BUT it is important to pick one method for yourself and stick to it from problem to problem. Pretend you are a computer; it will help eliminate errors. Here is the algorithm *I* use.

Method: In general the method involves trying to systematically transform the columns of A into *pivot* columns or *free* columns. We work on the columns starting on the leftmost one and work our way right. We keep track of the # of pivot columns that have been created up to this point. Lets call this number "p". When we start out, we work on the leftmost column and we have made no pivot columns, so p=0.

Suppose we are working on a particular column and we have already produced p pivots.

1) Check if all the entries of the column below the pth component are zero. We make a judgment call!

If Yes: Then we declare the column to be *free* and we move on to the next column. Since we have not made any more pivot columns, p stays the same. Go to the beginning of step **1)** using the next column and the same p.

For instance, if we are working on the second column of

$$\begin{bmatrix} 1 & 2 & 3 & 7 & 8 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 4 & 0 & 1 \end{bmatrix}$$

at this point p=1, and sure enough the second column is $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$, all entries past the first entry are zero, so we conclude the column is free.

If No: Then column will soon become the (p+1)st *pivot* using rules i),ii),iii) in step **2)**. For instance if are working on the third column of

$$\begin{bmatrix} 1 & 2 & 3 & 7 & 8 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 4 & 0 & 1 \end{bmatrix}$$

then again at this point p=1. The third column $\begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}$ has a nonzero entry below the 1st component (the 4). We conclude the column should be made into a pivot. Move onto **2)**.

2) We break up the process into three steps

a) If necessary, use rule i) to swap rows so that the (p+1)st entry of our column is nonzero (unlike in our example there may be a number of possible swaps that would work here).

In our example, swap the second and third row $\begin{bmatrix} 1 & 2 & 3 & 7 & 8 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 4 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 7 & 8 \\ 0 & 0 & 4 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

b) If necessary, use rule ii) to scale the (p+1)st row so that the (p+1)st entry of our column is now a 1.

In our example, scale the second row by $\dfrac{1}{4}$ $\begin{bmatrix} 1 & 2 & 3 & 7 & 8 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 4 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 7 & 8 \\ 0 & 0 & 1 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

c) If necessary, use rule iii) repeatedly to convert <u>*all*</u> of the other entries in our column to 0.

In our example, add -3×row 2 to row 1 $\begin{bmatrix} 1 & 2 & 3 & 7 & 8 \\ 0 & 0 & 1 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 0 & 7 & \frac{29}{4} \\ 0 & 0 & 1 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

d) Now our column is a *pivot* column. Since we have created another pivot column, we need to add 1 to p, so it becomes p+1. We move on to work on the next column. Go to the beginning of step **1)**.

Using the example above, p is now equal to 2 and we start work on the 4th column.