1 Helpful Facts About Linear Independence

You really, really, really need to know the definition of linear independence. One incarnation of the definition is

Definition 1.1. *A set of vectors* $\{\vec{v}_1, ..., \vec{v}_n\}$ *is linearly independent if the only solution to the equation* $c_1\vec{v}_1 + \cdots + c_n\vec{v}_n$ *is for all the coefficients to be zero, i.e.* $c_1 = \cdots = c_n = 0$.

Definition 1.2. *We say a collection of vectors is linearly dependent if it is not linearly independent.*

Here are some basic facts about linear independence that are important to know and good exercises to verify (this means "please try them and see me if you're having trouble").

1) $\{\vec{v}\}$ is linearly independent if and only if $\vec{v} \neq \vec{0}$.

2i) $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent if and only if $\vec{v}_1$ is a nonzero multiple of $\vec{v}_2$ and are not both the zero vector. WARNING! This is NOT true for collections of three or more vectors as you will see next.

2ii) Find an example of a collection of three nonzero vectors that are not multiples of one another, but are linearly dependent nonetheless.

2iii) No matter how large the collection of vectors, if two of them are multiples of one another show that the collection is linearly dependent.

3) Try to show that if a collection of vectors $\{\vec{v}_1, ..., \vec{v}_n\}$ is linearly independent and $\vec{v}_{n+1}$ is NOT in the span of $\{\vec{v}_1, ..., \vec{v}_n\}$, then the collection $\{\vec{v}_1, ..., \vec{v}_n, \vec{v}_{n+1}\}$ is linearly independent also.

Come midterm time, you'll want these problems to be as easy and enjoyable as... cleaning your ears. Come see me if you're getting stuck on them.