

Open book, open notes. Show all your work.

Name: _____

Acknowledgement and acceptance of the honor code:

Signature: _____

1. Let C be the circle $|z| = 2$, traversed counterclockwise. Evaluate the integrals

$$(5\text{points}) \quad I = \int_C \frac{\sin(4z)}{z} dz;$$

$$(5\text{points}) \quad I = \int_C \frac{\sin(4z)}{z^2} dz;$$

$$(5\text{points}) \quad I = \int_C \frac{dz}{z^2 + z + 5/4},$$

Clarify which theorem(s) you use and why they apply.

2. (10 points) Calculate the value of the following integral:

$$I = \int_0^{2\pi} e^{e^{i\theta} - i\theta} d\theta.$$

3. (a) (10 points) Find the first three terms in the Taylor series expansion about $z_0 = 0$ of the function

$$f(z) = \tanh z.$$

(b) (5 points) What is the radius of convergence of the above series?

4. (a) (10 points) Derive all different Taylor / Laurent series expansions about $z_0 = 0$ of the function

$$f(z) = \frac{z + 3}{z^2 + 9}.$$

(b) (5 points) What is the domain of convergence of each of the series?

_____ See second page _____

5. (a) (5 points) For what value of λ is there an analytic function whose real part is

$$u(x, y) = 6x - x^3 + \lambda xy^2?$$

- (b) (10 points) Find the corresponding harmonic conjugate function $v(x, y)$.