Let $M$ be a smooth $n$-manifold, $n \geq 1$. (I.e. $M$ is a topological manifold equipped with a particular smooth structure $[A]$ which we omit from the notation.) In this exercise we prove the equivalence of three possible definitions of “orientation”. For the purpose of distinguishing the concepts throughout the exercise we will use the names “A-orientation”, “B-orientation” and “C-orientation”.

**Definition A**: An $A$-orientation of $M$ is an orientation $\mu$ of the underlying topological manifold. $\mu$ is a family of “local orientations”, i.e. generators $\mu_x \in H_n(M, M - \{x\})$. They are required to satisfy a continuity condition, cf. Hatcher.

**Definition B**: An atlas $A$ in the smooth structure of $M$ is oriented if $\det(D(h' \circ h^{-1})(x)) > 0$ for all $h, h' \in A$ and all $x$ for which $h' \circ h^{-1}(x)$ is defined. Two oriented atlasses $A, A'$ are equivalent if the union $A \cup A'$ is again an oriented atlas. A $B$-orientation on $M$ is an equivalence class of oriented atlasses.

**Definition C**: A $C$-orientation on $M$ is a form $\omega \in \Omega^n(M)$ such that $\omega(x) \neq 0 \in \text{Alt}^n(T_xM)$ for all $x \in M$.

Note: I used definition B in the lectures. Madsen and Tornehave uses definition C.

1. Given an A-orientation $\mu$ of $M$, we let $A_{\text{max}}$ be the maximal atlas for the smooth structure. Let

$$A(\mu) = \{(h, U, U')| h : U \rightarrow U' \text{ preserves local orientations}\}$$

Prove that $\mu \mapsto A(\mu)$ gives a bijection between the set of A-orientations of $M$ and the set of B-orientations.

2. Construct a bijection between the set of B-orientations and the set of C-orientations on $M$. 