1. Let \textbf{Groups} be the category of (not necessarily commutative) groups, and \textbf{Sets} the category of sets. Let \( n \) be a natural number and let \( F : \textbf{Groups} \to \textbf{Sets} \) denote the functor
\[
F(G) = G^n = G \times \cdots \times G.
\]
(and on a morphism \( f : G \to H \), it’s given by \( F(f) = f^n : G^n \to H^n \)).

(i) Classify natural transformations \( T : F \to F \).

(ii) The set of natural isomorphisms \( T : F \to F \) is a group (natural transformations can be composed, cf. the note about Yoneda’s lemma). Which group is it?

(iii) Let \( F' : \textbf{Groups} \to \textbf{Groups} \) be the functor given by the same formula as \( F \), but now regarded as a functor into the category of groups. What is the group of natural isomorphisms \( F' \to F' \)?

2. The proof of theorem III.16.1 in Bredon (the construction of cross products) involves a lot of choices. This exercise studies some uniqueness properties of the cross product. We will use greek letters instead of “\( \times \)” for cross products in this exercise. Let us say that \( \mu \) is a cross product, if \( \mu : \Delta_p(X) \times \Delta_q(Y) \to \Delta_{p+q}(X \times Y) \) satisfies properties (1), (2) and (3) in Theorem 16.1

(i) Prove that if \( \mu \) and \( \nu \) are cross products, then there is a bilinear natural transformation \( T : \Delta_p(X) \times \Delta_q(Y) \to \Delta_{p+q+1}(X \times Y) \) such that
\[
\mu(a, b) - \nu(a, b) = \partial T(a, b) + T(\partial a, b) + (-1)^p T(a, \partial b).
\]

(ii) Prove that the homology cross product (Bredon proposition IV.16.3) is independent of the choice of cross product.

3. Let \( G \) be the space from the first problem on the first midterm.

(i) Use the CW structure to calculate \( \pi_1(G) \).

(ii) Prove that, in the notation of Bredon chapter IV.10, all the numbers \([\tau : \sigma]\) are even.

(iii) Calculate \( H_*(G; \mathbb{F}_2) \).