This is the corrected version of the first problem.

Rules: If you have questions about the exam, you may e-mail me or ask me in person. Discussing the problems with anyone else (before 5PM on 2/17) will be a violation of the Stanford Honor Code. Please hand in your solutions to me in person, either in class or in my office. Solutions received after the deadline will not be considered.

Name:

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Signature, acknowledgement of Honor Code:

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1. Let \( V \) denote the space of \( 2 \times 4 \)-matrices of rank 2, topologized as a subset of \( \mathbb{R}^8 \).

(a) Let \( \text{Gl}_2(\mathbb{R}) \) act on the left on \( V \) by matrix multiplication. Let \( G = \text{Gl}_2(\mathbb{R}) \setminus V \), equipped with the quotient topology. Show that \( G \) is compact and Hausdorff.

(b) Define functions \( \phi_{01}, \phi_{02}, \phi_{11}, \phi_{12} \) and \( \phi_{22} \) in the following way

\[
\text{int}(D^1) \xrightarrow{\phi_{01}} V \\
(1 \ 0 \ 0 \ 0) \\
0 \ t \ \sqrt{1 - t^2} \ 0
\]

\[
\text{int}(D^2) \xrightarrow{\phi_{02}} V \\
(1 \ 0 \ 0 \ 0) \\
0 \ t_1 \ t_2 \ \sqrt{1 - \|t\|^2}
\]

\[
\text{int}(D^1) \times \text{int}(D^1) \xrightarrow{\phi_{11}} V \\
(s, t) \mapsto \begin{pmatrix} s \sqrt{1 - s^2} & 0 & 0 \\ t \frac{s^2}{\sqrt{1 - s^2}} & \sqrt{1 - t^2} & 0 \end{pmatrix}
\]

\[
\text{int}(D^1) \times \text{int}(D^2) \xrightarrow{\phi_{12}} V \\
(s, t) \mapsto \begin{pmatrix} s \sqrt{1 - s^2} & 0 & 0 \\ t_1 \frac{s t}{\sqrt{1 + |s|^2}} & t_2 & \sqrt{1 - \|t\|^2} \end{pmatrix}
\]

\[
\text{int}(D^2) \times \text{int}(D^2) \xrightarrow{\phi_{22}} V \\
(s, t) \mapsto \begin{pmatrix} s_1 \ s_2 \\ t_1 \ t_2 \end{pmatrix} \begin{pmatrix} \sqrt{1 - \|s\|^2} \\ \frac{-s t}{\sqrt{1 + |s|^2}} \\ \sqrt{1 - \|t\|^2} \end{pmatrix}
\]

Prove that the compositions \( \text{int}(D^i) \times \text{int}(D^j) \to V \to G \) extend to a continuous maps

\[
\varphi_{ij} : D^i \times D^j \to G.
\]

(c) Using a homeomorphism \( D^i \times D^j \approx D^{i+j} \), we get maps \( \sigma_{ij} : D^{i+j} \to G \). Prove that there is a CW structure on \( G \) with cells of dimension 0, 1, 2, 2, 3, 4, where the maps \( \sigma_{ij} \) are characteristic maps of the cells of positive dimension.

(d) What would happen if we start instead with \( V \) being the space of \( 2 \times 5 \) matrices of rank 2? (You do not need to repeat arguments that are essentially the same as in the case \( 2 \times 4 \).)