Here are some suggestions for what you might incorporate in your wim text. Remember, the focus is on the exposition, and part of making a good exposition is making the right choice of content. You should first read sections 69, 80 and 81 in the book, and make sure you can do problem 82.6.

You should imagine that you are writing a chapter in a text book. The target audience for your text is a student in a class similar to 171. You should assume that your audience already knows more basic definitions (what is a metric space, what is a vector space, what is a Cauchy sequence, etc). Your text should be short, 4–7 pages, and contain a brief introduction, the relevant theory, and some application/example. Here are some more concrete suggestions for what you could cover (if you wish to stray away from those, you should discuss it with me or Shotaro first).

1. Introduce normed vector spaces and their relation to metric spaces. Discuss the space $\mathcal{L}(V,W)$ of continuous linear maps $V \to W$ and prove that it is again a normed vector space if $V$ and $W$ are. Your text could incorporate solutions to some of the exercises if you think they’re relevant, e.g. 81.5. (But of course you shouldn’t phrase this as “solutions to exercise in book” – you’re writing a chapter for another book.)

2. Discuss how $m \times n$ matrices give an example: $M_{n \times m}(\mathbb{R}) = \mathcal{L}(\mathbb{R}^m, \mathbb{R}^n)$.

3. Discuss completeness (cf §46, we haven’t gotten there yet (5/2) but will very soon) and prove that $\mathcal{L}(V,W)$ is complete if $W$ is. (We’ll prove in class that $\mathbb{R}^n$ is complete.)

4. An interesting class of examples which you could discuss come from power series. For example, if $A \in M_n(\mathbb{R})$ (i.e. $A$ is a square $n \times n$ matrix), you could consider the matrices

$$S_n = I + A + A^2 + \cdots + A^n$$

and prove that they form a Cauchy sequence in $M_n(\mathbb{R})$ if $\|A\| < 1$. What does it converge to?