Axler, Chapter 7: 1, 3, 5, 6, 7, 9, 11, and these exercises.

8. Let $V$ be a finite dimensional complex inner product space.

(i) Prove that any $T \in \mathcal{L}(V)$ can be written as $T = S_1 + iS_2$ for unique self-adjoint operators $S_i \in \mathcal{L}(V)$. (Hint: first prove uniqueness.)

(ii) Prove that $T$ is normal if and only if $S_1S_2 = S_2S_1$.

9. Let $T \in \mathcal{L}(V)$ be normal, and let $S_1$ and $S_2$ be as in the previous exercise. Prove that $v \in V$ is an eigenvector for $T$ if and only if it is an eigenvector for both $S_1$ and $S_2$. 