Unless explicitly noted otherwise, you must justify your answers. (A proper justification almost always involves words and whole sentences, not just a string of formulas.)

Axler 1.3, 1.5, 1.6, 1.9, 1.10, 1.11

1. In this exercise (and only this one!), you do not need to justify your answers. As mentioned in class, when dealing with a vector space $V$ over a field $\mathbb{F}$, the symbols “$+$” and “$0$” are potentially ambiguous, since they have one meaning in $\mathbb{F}$ and one meaning in $V$.

Read the statements and proofs of Propositions 1.4, 1.5, and 1.6 in the book. Each proof has an equation. Copy the equation and indicate for each occurrence of “$+$” and “$0$” whether Axler intends the meaning in $\mathbb{F}$ or the one in $V$.

2. Let $\mathbb{F}$ be a field and let $V$ and $W$ be vector spaces over $\mathbb{F}$. Let $V \times W$ denote the set of ordered pairs $(v, w)$ with $v \in V$ and $w \in W$. For $a \in \mathbb{F}$ and elements $x_1 = (v_1, w_1)$ and $x_2 = (v_2, w_2)$ of $V \times W$, define

$$x_1 + x_2 = (v_1 + v_2, w_1 + w_2)$$

$$ax_1 = (av_1, aw_1)$$

Prove that $V \times W$ is a vector space with these operations.

3. Let $\mathbb{F}$ be a field, $V$ a vector space over $\mathbb{F}$, and let $X$ be any set. Let us write $V^X$ for the set of functions $X \to V$. For $f, g \in V^X$ and $a \in \mathbb{F}$, define $f + g \in V^X$ and $af \in V^X$ by

$$(f + g)(x) = f(x) + g(x)$$

$$(af)(x) = a(f(x)).$$

Prove that these definitions make $V^X$ into a vector space.

4. Prove that the only subspaces of $\mathbb{F}^1$ are $\{0\}$ and $\mathbb{F}^1$.

5. Let $F$ be $\mathbb{C}$ or $\mathbb{R}$. For each $a \in \mathbb{F}$, define a subset $V_a \subseteq \mathbb{F}^2$ by

$$V_a = \{(x_1, x_2) \mid x_1 = a(x_2)^2\}.$$ 

For which $a$ is $V_a$ a subspace of $\mathbb{F}^2$?