Math 106, Mid-term 2, November 17, 2004

Open book, open notes. All problems have equal weight.

Name: ________________________________

Acknowledgement and acceptance of honor code:

Signature: ________________________________

1. Let $C$ be the contour which is a straight line from $i+1$ to $i-1$, parametrized in that direction (i.e. from right to left).

   (a) Sketch $C$ and compute
   \[ \int_C z^2 \, dz \]

   (b) Prove that
   \[ \left| \int C e^z \, dz \right| \leq 2 \]

2. Let $C$ be the circle $|z| = 2$, parametrized counterclockwise.

   (a) Compute
   \[ \int_C (e^z + e^{-z}) \, dz \]

   (b) Compute
   \[ \int_C \frac{e^z + e^{-z}}{z} \, dz \]

   Clarify which theorem(s) you use and why they apply.

3. Let $f(z) = \frac{1}{z^6 + z^2}$.

   (a) Let $D$ be the domain $|z| > 2$. Prove that $|f(z)|$ has no maximum in $D$.

   (b) Let $C$ be the square with corners $\pm 1$ and $\pm i$. Compute
   \[ \int_C f(z) \, dz \]

   Clarify which theorem(s) you use and why they apply.