1. Let $C$ be the circle $|z| = 2$, traversed counterclockwise. Evaluate the integrals

\[ I = \int_C \frac{\sin(4z)}{z} \, dz; \]

\[ I = \int_C \frac{\sin(4z)}{z^2} \, dz; \]

\[ I = \int_C \frac{dz}{z^2 + z + 5/4}. \]

Clarify which theorem(s) you use and why they apply.

2. (10 points) Calculate the value of the following integral:

\[ I = \int_0^{2\pi} e^{e^{i\theta} - i\theta} \, d\theta. \]

3. (a) (10 points) Find the first three terms in the Taylor series expansion about $z_0 = 0$ of the function

\[ f(z) = \tanh z. \]

(b) (5 points) What is the radius of convergence of the above series?

4. (a) (10 points) Derive all different Taylor / Laurent series expansions about $z_0 = 0$ of the function

\[ f(z) = \frac{z + 3}{z^2 + 9}. \]

(b) (5 points) What is the domain of convergence of each of the series?
(a) (5 points) For what value of $\lambda$ is there an analytic function whose real part is

$$u(x, y) = 6x - x^3 + \lambda xy^2.$$ 

(b) (10 points) Find the corresponding harmonic conjugate function $v(x, y)$. 
