Here is the text for the exercises due Friday 10/7.

page 11, # 4: In each case, sketch the set of points determined by the given condition: (a) $|z - 1 + i| = 1$, (b) $|z + i| \leq 3$, (c) $|z - 4i| \geq 4$.

page 13, # 2: Sketch the set of points determined by the condition (a) Re($\overline{z} - i$) = 2, (b) $|2z - i| = 4$.

page 21, # 1: Find the principal argument Arg($z$) when (a) $z = i - 2 - 2i$, (b) $z = (\sqrt{3} - i)^6$. Ans.: (a) $-\frac{3}{4}\pi$, (b) $\pi$.

page 21, # 10: Establish the identity

$$1 + z + z^2 + \ldots + z^n = \frac{1 + z^{n+1}}{1 - z} \quad (z \neq 1)$$

and then use it to derive Lagrange’s trigonometric identity:

$$1 + \cos \theta + \cos 2\theta + \ldots + \cos n\theta = \frac{1}{2} + \frac{\sin[(2n + 1)\theta/2]}{2\sin(\theta/2)} \quad (0 < \theta < 2\pi)$$

Suggestion: As for the first identity, write $S = 1 + z + z^2 + \ldots + z^n$ and consider the difference $S - zS$. To derive the second identity, write $z = e^{i\theta}$ in the first one.

page 28, # 2: In each case, find all the roots in rectangular coordinates, exhibit them as vertices of certain squares, and point out which is the principal root: (a) $(-16)^{1/4}$, (b) $(-8 - 8\sqrt{3})^{1/4}$. Ans: (1) $\pm\sqrt{2}(1 + i)$, $\pm\sqrt{2}(1 - i)$, (b) $\pm(\sqrt{3} - i)$, $\pm(1 + \sqrt{3}i)$.

page 28, # 6: Find the four roots of the equation $z^4 + 4 = 0$ and use them to factor $z^4 + 4$ into quadratic factors with real coefficients. Ans: $(z^2 + 2z + 2)(z^2 - 2z + 2)$.

page 31, # 1: Sketch the following sets and determine which are domains: (a) $|z - 2 + i| \leq 1$, (b) $|2z + 3| > 4$, (c) Im$z > 1$, (d) Im$z = 1$, (e) $0 \leq \arg z \leq \pi/4$ ($z \neq 0$), (f) $|z - 4| \geq |z|$. Ans: (b), (c) are domains.

page 31, # 4: In each case, sketch the closure of the set: (a) $-\pi < \arg z < \pi$ ($z \neq 0$), (b) Re$z < |z|$, (c) Re$(1/z) \leq 1/2$, (d) Re$(z^2) > 0$.

page 35, #2: Write the function $f(z) = z^3 + z + 1$ in the form $f(z) = u(x, y) + iv(x, y)$. Ans. $(x^3 - 3xy^2 + x + 1) + i(3x^2y - y^3 + y)$.

page 35, #4: Write the function $f(z) = z + \frac{1}{z}$, ($z \neq 0$), in the form $f(z) = u(r, \theta) + iv(r, \theta)$. Ans. $(r + \frac{1}{r})\cos \theta + i(r - \frac{1}{r})\sin \theta$.

page 42, # 2: Find and sketch, showing corresponding orientations, the images of the hyperbolas $x^2 - y^2 = c_1$ ($c_1 < 0$) and $2xy = c_2$ ($c_2 < 0$).

page 42, # 7: Find the image of the semi-infinite strip $x \geq 0$, $0 \leq y \leq \pi$ under the transformation $w = \exp z$, and label corresponding portions of the boundaries.