

# Stanford University Department of Mathematics

## Math 42 — Second Midterm

Examiner: Adrian Butscher

Date: 26 February 2008

Duration: 120 minutes

FAMILY NAME: \_\_\_\_\_

GIVEN NAME(S): \_\_\_\_\_

STUDENT NUMBER: \_\_\_\_\_

THE TIME OF YOUR DISCUSSION SECTION: \_\_\_\_\_

THE NAME OF YOUR TA: \_\_\_\_\_

Jarod Alper

Dung-Huang Nguyen

Yu-Jong Tzeng

Ian Weiner

YOUR SIGNATURE: \_\_\_\_\_

**DO NOT OPEN THIS TEST UNTIL INSTRUCTED TO DO SO.**

### INSTRUCTIONS:

- Your signature above indicates that you have abided by the Stanford Honour Code while writing this test.
- All questions have equal value (20 points). There are six questions.
- You may quote theorems from your textbook if you make an appropriate reference.
- Show all your work.
- No aids of any kind (e.g. textbooks, notes, calculators, cell-phones) are allowed.

Question	Marks
1	
2	
3	
4	
5	
6	
Total (120 points)	

1. A cargo plane flies at height  $h$  above the ground with speed  $v$ . At time  $t = 0$  the cargo doors open and an item is pushed out of the cargo bay, whence it plummets to the ground. Let  $x$  denote a coordinate in the direction of motion of the plane and let  $z$  denote the height coordinate, with the ground located at  $z = 0$ . Assume that the trajectory of the item satisfies the differential equation

$$\begin{aligned}z'' &= -g \\x'' &= 0\end{aligned}$$

with initial conditions  $x(0) = 0$ ,  $x'(0) = v$ ,  $z(0) = h$  and  $z'(0) = 0$ .

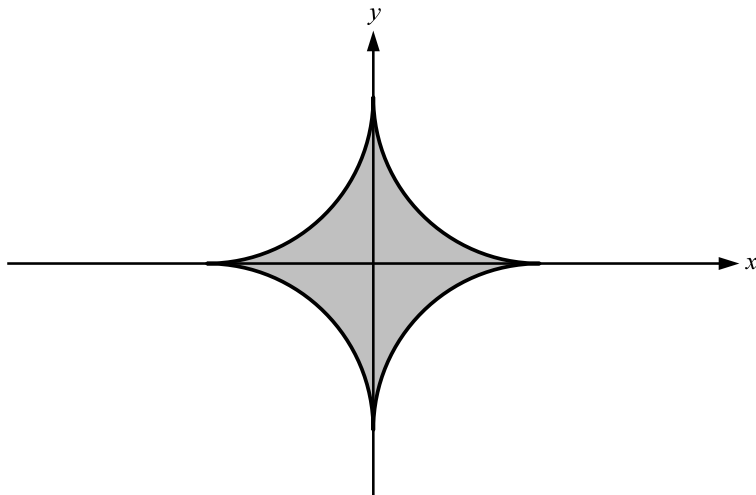
- (a) When does the item hit the ground?

- (b) Where does the item hit the ground?

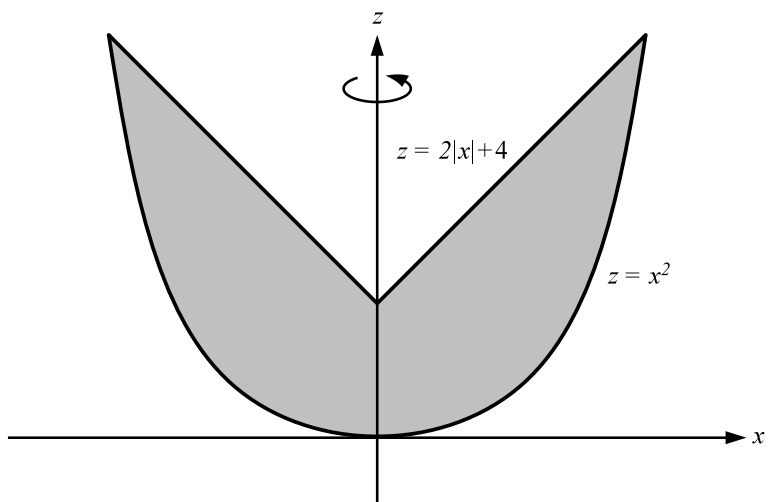
(c) What is the velocity of the object when it hits the ground?

2. Find the length of the curve  $y = \frac{1}{6}(x^2+4)^{3/2}$  for  $x \in [0, 3]$ . (Hint:  $4+4y+y^2 = (y+2)^2$ .)

3. The height of a monument is  $h$ . The shape of its horizontal cross-section at height  $z$  is given in the following picture, where each of the four curves is one quarter of a circle of radius  $h - z$ . Find the volume of the monument.



4. Let us generate a solid of revolution by rotating the shaded region pictured below about the  $z$  axis.



- (a) Write down (but do not evaluate) an integral for the volume of the solid using the method of washers/disks.

- (b) Write down (but do not evaluate) an integral for the volume of the solid using the method of shells.

5. Find all solutions of the ordinary differential equation

$$y'''(t) = \cos(t).$$

6. Find a solution of the ordinary differential equation

$$y'(t) = \frac{te^{[y(t)]^2}}{y(t)}$$

satisfying the initial condition  $y(0) = 1$  .

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