Stanford University Department of Mathematics

Math 42 — First Midterm

Examiner: Adrian Butscher

Date: 17 October 2007

Duration: 120 minutes

Family Name: _____

Given Name(s): _____

Student Number: _____

The time of your tutorial: _____

The name of your TA: _____

Theodora Bourni Ken Chan Ben Williams

Your signature: _____

DO NOT OPEN THIS TEST UNTIL INSTRUCTED TO DO SO.

Instructions:

- Your signature above indicates that you have abided by the Stanford Honour Code while writing this test.
- All questions have equal value (20 points). There are six questions and a bonus.
- You may quote theorems from your textbook if you make an appropriate reference.
- Show all your work.
- No electronic devices of any kind (e.g. calculators, cell-phones) are allowed.

Question	Marks
1	
2	
3	
4	
5	
6	
Bonus	
Total (120 points)	

1. Find the value of the sum $\lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \left(1 + \frac{k}{N}\right)^5$ by expressing it as a definite integral.

2. Suppose $\int_0^1 f(x)dx = 1$ and $\int_0^3 f(2x)dx = 4$. Find $\int_0^5 f(x+1)dx$.

3. Find the derivative of $\int_{x^2}^{x^3} \sin(t)dt$.

4. Compute the integral

$$\int x^3 \sin(x^2) dx \,.$$

(Hint: substitution *then* integration by parts.)

5. Compute the integral

$$\int \sin(x)\cos(3x)dx\,.$$

(Hint: $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$ and $\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$.)

Question 5 — continued.

6. Compute the integral $\displaystyle\int \frac{1}{(x-2)(x+4)^2}dx$.

Bonus. Compute the integral $\displaystyle\int \frac{1}{(x^2+1)^2}dx$.

This page has been left blank for your rough work.