# Math 177: Homework N1 

Due on Monday, April 23

1. Consider a system of two quasi-homogeneous equations

$$
\begin{aligned}
& x y \frac{d y}{d x}=z^{2} \\
& z \frac{d z}{d x}=x y, \quad x, y, z>0 .
\end{aligned}
$$

Find a change of variables which reduces it to a first order system.
2. A particle is moving in a central potential field $V(x)=C\|x\|^{\alpha}$ in $\mathbb{R}^{3}$. Suppose that there exist a homothety $x \mapsto \lambda x, x \in \mathbb{R}^{3}$, which maps a periodic orbit $\Gamma_{1}$ onto a periodic orbit $\Gamma_{2}$. Suppose that the periods of the orbits are $T_{1}$ and $T_{2}$, respectively. Find $\alpha$.
3. Compute the phase flow of the vector field on the plane

$$
v=x \frac{\partial}{\partial y}+y \frac{\partial}{\partial x} .
$$

Suppose that a line field

$$
\ell:=\{a(x, y) d x+b(x, y) d y=0\}
$$

is invariant under this flow. Which change of variables one needs to make in order to solve the equation

$$
a(x, y) d x+b(x, y) d y=0
$$

4. Find the solutions, the criminant and the discriminant of the equation

$$
\left(y^{\prime}\right)^{3}+3 x y^{\prime}-3 y=0
$$

5. Let $f=\sum_{1}^{n} a_{i j} x_{i} x_{j}$ be a quadratic form on $\mathbb{R}^{n}$. Show that its Legendre transform $g(p)$ is again a quadratic form $g(p)=\sum_{1}^{n} b_{i j} p_{i} p_{j}$ and that the value of these forms at the points corresponding to each other under the Legendre map coincide.
(Legendre map is a diffeomorphism $x \mapsto p$ which is used in the definition of Legendre transform.)

Each problem is 10 points.

