Math 137. Mathematical Methods of Classical Mechanics

Winter 2013

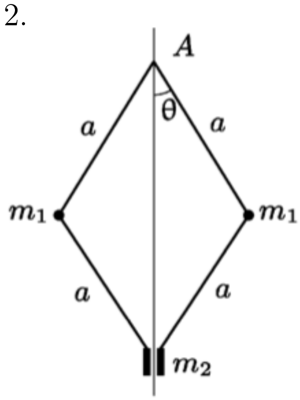
Homework N1; Due to Monday, January 28

1. Let $Q(x)$ be a positive definite quadratic form on $\mathbb{R}^n$. Consider two functionals on the space of paths $\gamma : [0,1] \to \mathbb{R}^n$ connecting two points $A, B \in \mathbb{R}^n$,

$$S_1(\gamma) = \int_0^1 Q(\dot\gamma(t))dt \text{ and } S_2(\gamma) = \int_0^1 \sqrt{Q(\dot\gamma(t))}dt,$$

where $\dot\gamma = \frac{d\gamma}{dt}$. Compare solutions of Euler-Lagrange equations for two functionals.

2.



In the system shown on Fig. the mass m_2 moves along the vertical axis and the whole system rotates with the angular velocity Ω around the vertical axis. Assuming that the system is in the constant field of gravity g write the Lagrange function of the system.

3. At the entree of a satellite into a circular orbit at a distance 300km from the Earth the actual velocity was 1 m/sec less than it was intended (in order for the orbit to be a circular one). How does the height of the pericenter change?
Assume Earth radius $= 6.3 \cdot 10^6$m.

4. The center of the Earth describes an ellipse around the common center of mass of the system (Earth, Moon). Determine the period of this orbit. If m

and M denote the masses of the Moon and the Earth, respectively, assume that $M = 81m$.

5. Consider two equations

$$\ddot{\phi} = -g \sin \phi, \tag{1}$$

and

$$\ddot{\phi} = -g\phi, \tag{2}$$

where $g = 9.8\, m/sec^2$. The first equation describes a motion of a pendulum of length 1 in the field of gravity, while the second one is its linearization (desribing small oscillations). Here ϕ is the angle of the deviation of the pendulum from the vertical line. One originally lifts the pendulum to the angle $\phi = 1^\circ$ and then lets it go. Find the difference between the periods of oscillation of the two systems. *Hint*: write the Taylor expansion of $\sin \phi$.

6. Find explicitly the trajectory of a particle (of mass 1) in a central field with a potential energy $U(r) = -\frac{\alpha}{r^2}$, $\alpha > 0$ and describe the motion. Determine the values of the energy E and the angular momentum M for which the particle reaches the center in a finite time.

7. Find and study behavior of all geodesics on the torus given in cylindrical coordinates (z, r, ϕ) by an equation

$$(r - R)^2 + z^2 = \rho^2, \quad \rho < R.$$