Math 53H: Homework N7

Due to Friday, May 25

1. Find the general form of a real solution of the equation

$$\ddot{x} + x = 5te^{-2t} + 4\sin t.$$

2. Find all real values a, α, β for which the initial value problem

$$\ddot{x} - 2\dot{x} + 5x = ae^t \cos 2t - 17 \sin 2t, \;\; x(0) = \alpha, \; \dot{x}(0) = \beta,$$

has a bounded solution when $t \to +\infty$?

3. Use the method of variation of constants to find any particular solution of the system

$$\dot{x} = 3x - 2y,$$
$$\dot{y} = 2x - y + 15e^t\sqrt{t}.$$

4. Consider the system

$$\dot{x} = -\sin y,$$
$$\dot{y} = \sin x + \sin y.$$

Find all its equilibrium points and study their stability.

5. Find all values of the parameter a for which the origin is an asymptotically stable equilibrium point for the system

$$\dot{x} = ax - 2y + x^2,$$
$$\dot{y} = x + y + xy.$$

6. Find a necessary and sufficient condition for systems

$$\dot{x} = -\omega y,$$
$$\dot{y} = \omega x.$$

and

$$\dot{x} = -\nu y,$$
$$\dot{y} = \nu x.$$

to be topologically equivalent.

Each problem is 10 points.