

Math 53H: Homework N6

Due to Friday, May 18

1. It is known that a system

$$\dot{x} = Ax,$$

$x = (x_1, \dots, x_n) \in \mathbb{R}^n$ has a solution $x(t)$ such that $x_1(t) = t^2 + t \sin t$. Prove that $n \geq 7$.

2. Consider the equation

$$\ddot{x} - x = 1.$$

Find the solution $x(t)$ which is bounded when $t \rightarrow +\infty$ and which satisfies $x(0) = 0$.

3. Find all real values a for which the equation

$$\ddot{x} + ax = 1$$

has no real solutions such that $x(0) = x(1) = 0$.

4. Find a general solution of the system

$$\dot{x} = 4x - y,$$

$$\dot{y} = 3x + y - z,$$

$$\dot{z} = x + z$$

The system has a unique eigenvalue 2 of multiplicity 3.

5. Find a necessary and sufficient condition for the Jordan form of a matrix A to satisfy the identity

$$e^{tA} = I + tA + 2t^2 A^2 + 6t^3 A^3 + 24t^4 A^4 + 120t^5 A^5.$$

Each problem is 10 points.