Math 53H: Homework N5

Due to Monday, May 14

1. Prove, that for any $n \times n$ matrices A, B one has

$$\frac{\partial^2}{\partial s \partial t}\left(e^{As}e^{Bt}e^{-As}e^{-Bt}\right) = AB - BA.$$

2. Find all complex matrices A which are similar only to itself, i.e. such that if $B = C^{-1}AC$ for some C then $B = A$.

3.Let $\mathcal{A} : V \to V$ be an invertible operator. Prove that the characteristic polynomial $\chi_{\mathcal{A}^{-1}}(\lambda)$ of the operator $\mathcal{A}^{-1}$ is proportional to $\lambda^n \chi_{\mathcal{A}}(\frac{1}{\lambda})$.

4. Denote

$$J_m := \begin{pmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ & \dots & \dots & \dots & \dots & \\ 0 & 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & 0 & \dots & 0 & 0 \end{pmatrix}.$$

Find the Jordan normal form of the matrix J_m^2.

5. Given a matrix

$$A = \begin{pmatrix} 0 & -2 & 3 & 2 \\ 1 & 1 & -1 & -1 \\ 0 & 0 & 2 & 0 \\ 1 & -1 & 0 & 1 \end{pmatrix},$$

find matrices C and B such that $A = CBC^{-1}$ and B is in a Jordan normal form.

Each problem is 10 points.