

Math 53H: Homework N4

Due to Monday, May 7

Number and sections are given according to the book
V. I. Arnold, Ordinary Differential Equations, 3rd Edition

1. Find an example of two vector fields on \mathbb{R} : $X = f(x)\frac{d}{dx}$ and $Y = g(x)\frac{d}{dx}$ such that all the Lie brackets

$$[X, Y], [[X, Y], Y], \dots, [[\dots [[X, Y], Y], \dots, Y], Y]$$

coincide and do not equal to 0.

Hint: Take $f(x) = e^x$ and find a vector field Y such that $[X, Y] = X$.

2. Consider \mathbb{R}^4 with the form $\omega = dp_1 \wedge dq_1 + dp_2 \wedge dq_2$.

Find a change of coordinates

$$(p_1, q_1, p_2, q_2) \mapsto (P_1, Q_1, P_2, Q_2),$$

such that

$$\omega = dP_1 \wedge dQ_1 + dP_2 \wedge dQ_2 \text{ and}$$

$$Q_1 = q_1 + q_2, \quad Q_2 = q_1 - q_2.$$

3. Problem 2 in 2.12.9.

Each problem is 10 points.