Math 53H: Homework N2

Due to Friday, April 20

Number and sections are given according to the book

V. I. Arnold, Ordinary Differential Equations, 3rd Edition

1. Problem 8 in Section 1.4.4.

2. Problem 2 in Section 1.5.2.

3. Problem 3 in Section 1.6.5.

4. Consider differential equations

$$\begin{cases} \dot{x}_1 &= -(x_1^2 + x_2^2)x_2 \\ \dot{x}_2 &= (x_1^2 + x_2^2)x_1. \end{cases} \tag{1}$$

and

$$\begin{cases} \dot{x}_1 &= -x_2 \\ \dot{x}_2 &= x_1. \end{cases} \tag{2}$$

a) Find explicitly the phase flows of these equations.

b) Prove that there is no diffeomorphism $\mathbb{R}^2 \setminus 0 \to \mathbb{R}^2 \setminus 0$ between the vectors fields of the equation (1) and the equation (2).

c) However, prove that there exists a diffeomorphism between the corresponding line fields.

5. Consider the following system of differential equations:

$$\ddot{x} = -x$$
$$\ddot{y} = -a^2 y,$$

where a is an irrational number. Find all periodic solutions of this system and determine their periods.

Each (sub)problem is 10 points.