Math 53H: List of Topics

References given to sections of the book

V.I. Arnold, Ordinary Differential Equations

3rd edition, Universitext, Springer, 1992 (2006)

1. Vector fields as differential equations

(a) Phase space, integral curves, vector and line fields (Chapter 1, Sections 1,4,5)

(b) Vector fields on the line (Chapter 1, Section 2)

(c) Equations with separable variables (Chapter 1, Section 2)

(d) Examples (Chapter 1, Section 1)

(e) Linear equations (Chapter 1, Section 3)

(f) Equations with symmetries (Chapter 1, Section 6)

2. Main Theorems

(a) Rectification theorem for line and vector fields (Chapter 2, Section 7)

(b) Existence, uniqueness, dependence on initial conditions (Chapter 2, Section 7)

(c) Equations of higher order (Chapter 2, Section 8)

3. Autonomous systems

(a) The phase portrait of an autonomous system (Chapter 2, Section 9)

(b) Lie algebra of vector fields; first integrals (Chapter 2, Section 10)

(c) Conservative systems with one degree of freedom (Chapter 2, Section 12)

(d) Hamiltonian systems (Additional materials will be posted)

(e) First order partial differential equations (Chapter 2, Section 11)

4. Linear systems

(a) Linear vector fields and linearization (Chapter 3, Section 13)

(b) Exponential function of a linear operator (Chapter 3, Sections 14,15)

(c) The determinant of an exponential function (Chapter 3, Section 16)

(d) Complex vector spaces; complexification and de-complexification (Chapter 3, Section 18; additional material will be posted)

(e) Computation of an exponential function; case of discrete eigenvalues (Chapter 3, Section 17)

(f) Complexification of a real linear differential equation (Chapter 3, Section 20)

(g) Jordan normal form (Additional material will be posted)

(h) Computation of an exponential function in the case of multiple eigenvalues (Chapter 3, Section 25)

(i) Quasi-polynomials (Chapter 3, Section 26)

(j) Non-autonomous linear equations (Chapter 3, Section 27)

(k) Linear equations with periodic coefficients (Chapter 3, Section 28)

5. Stability theory

(a) Classification of singular points of linear systems (Chapter 3, Sections 21,22)

(b) Stability of equilibrium positions (Chapter 3, Section 23)

(c) Case of purely imaginary eigenvalues (Chapter 3, Section 24).

6. Proofs of the main theorems

(a) Metric spaces and contraction mapping theorem (Chapter 4, Sections 30,31; additional material will be posted)

(b) Picard's method of successive approximations; proof of the existence theorem (Chapter 4, Section 31)

(c) Proof of the rectification theorem (Chapter 4, Section 32)