1. Compute
\[ \int_{\gamma} xdy - ydx \]
with \( \gamma : [0, \pi] \rightarrow \mathbb{R}^2 \) given by
\[ \gamma(t) = \left( 1 + \frac{1}{2} \sin \frac{t^2}{\pi}, \cos \frac{t^3}{2\pi^2} \right). \]

2. Let \( A \subset V \) be a closed oriented manifold of dimension \( n = k + l + 1 \), \( \omega \) a differential \( k \)-form and \( \eta \) a differential \( l \)-form on \( V \). Prove that
\[ \int_A \omega \wedge d\eta = C \int_A \eta \wedge d\omega \]
for some constant \( C \), and find \( C \).

3. Let \( \Gamma \) be a closed oriented curve in \( \mathbb{R}^2 \) of length \( L \). Prove that
\[ \left| \int_{\Gamma} Pdx + Qdy \right| \leq L \max_{\Gamma} \sqrt{P^2 + Q^2}. \]
4. Compute the area of the surface

\[(x^2 + y^2)^{3/2} + z = 1, \ z \geq 0.\]

5. Using Stokes’ theorem compute the area of the domain in \(\mathbb{R}^2\) bounded by the astroid

\[x = a \cos^3 t,\]
\[y = b \sin^3 t,\]

where \(a, b > 0, \ 0 \leq t \leq 2\pi\).

6. Suppose that the circle \(S_R = \{x^2 + y^2 = R^2\} \subset \mathbb{R}^2\) is oriented counter-clockwise. Denote

\[I(R) := \int_{S_R} \frac{ydx - xdy}{(x^2 + xy + y^2)^2}.\]

Prove that \(I(R) \xrightarrow{R \to \infty} 0\).

6. Assuming that the Earth is a sphere of radius \(R = 6.3 \cdot 10^6 m\) compute the area bounded by the Tropic of Cancer, the Arctic Circle, Greenwich meridian and the 30°E meridian.

7. Using Stokes’ theorem compute the integral

\[\int_C (y + z)dx + (z + x)dy + (x + y)dz,\]

where \(C\) is the ellipse, defined parametrically by the formulas

\[x = a \sin^2 t,\]
\[y = 2a \sin t \cos t,\]
\[z = a \cos^2 t,\]

where \(t \in [0, \pi]\). The ellipse is oriented by the parameter \(t\).

Each problem is 10 points.