1. Denote coordinates in \( \mathbb{R}^2 \) by \((u, s)\) and in \( \mathbb{R}^4 \) with coordinates \((x_1, x_2, x_3, x_4)\). Consider a differential 1-form
\[
\alpha := x_4(dx_3 - x_2dx_1).
\]
Let \( h : \mathbb{R}^2 \to \mathbb{R} \) be a smooth function such that \( \frac{\partial h}{\partial s}(u, s) > 0 \) for all \((u, s) \in \mathbb{R}^2\). Find a smooth function \( g : \mathbb{R}^2 \to \mathbb{R} \) such that the form \( G^*\alpha \) on \( \mathbb{R}^2 \) is closed for the map \( G : \mathbb{R}^2 \to \mathbb{R}^4 \) given by the formula
\[
G(u, s) = \left( u, \frac{\partial h}{\partial u}(u, s), h(u, s), g(u, s) \right).
\]
Note that there are infinitely many functions \( g \) which satisfy this condition. You need to find just any of them.

*Hint.* First compute \( G^*\alpha \).

2. Given a function \( f : \mathbb{R}^n \to \mathbb{R} \), consider a map \( F : \mathbb{R}^n \to \mathbb{R}^{2n+1} \) defined by the formula
\[
F(x_1, \ldots, x_n) = \left( x_1, \ldots, x_n, \frac{\partial f}{\partial x_1}(x_1, \ldots, x_n), \ldots, \frac{\partial f}{\partial x_n}(x_1, \ldots, x_n), f(x_1, \ldots, x_n) \right).
\]
Compute \( F^*(\alpha) \), where
\[
\alpha = dx_{2n+1} - \sum_{i=1}^{n} x_{i+n}dx_i.
\]
3. On $\mathbb{C} \setminus 0$ consider complex-valued 1-forms
\[ \alpha_n = \frac{dz}{z^n} \quad \text{and} \quad \beta_n = \frac{d\bar{z}}{z^n}, \quad n \geq 1. \]

Compute $\int_{\Gamma} \alpha_n$ and $\int_{\Gamma} \beta_n$ where $\Gamma = \{z = e^{it}, \ t \in [0, 2\pi]\}$.

4. Prove that the 1-form
\[ \alpha := (x^2 + yz - y - z)dx + (y^2 + xz - x - z)dy + (z^2 + xy - x - y)dz \]

is exact and find its primitive, i.e a function $F : \mathbb{R}^3 \to \mathbb{R}$ such that $dF = \alpha$.

Problems are 10 points each.