Math 52H: Homework N6

Due to Friday, February 22

1. Prove that the submanifold $T = \{x_1^2 + x_2^2 = 1,\ x_3^2 + x_4^2 = 1\} \subset \mathbb{R}^4$ and the submanifold $U \subset \mathbb{R}^3$ given in cylindrical coordinates (r, ϕ, z) by the equation $(r-2)^2 + z^2 = 1$, are diffeomorphic.

2. Prove that the sets $A = \{xy(x+y) = 0\} \subset \mathbb{R}^2$ and $B = \{xy(x+2y)\} = 0\}$ are diffeomorphic, but the sets $C = \{xy(x+y)(x-y) = 0\} \subset \mathbb{R}^2$ and $D = \{xy(x+2y)(x-y) = 0\}$ are not.

3. Compute the integral

$$\iint\limits_D \left| \frac{x+y}{2} - x^2 - y^2 \right| dxdy,$$

where D is the unit disc $\{x^2 + y^2 \leq 1\}$.

4. Compute the integral

$$\iint\limits_{D_a} x^{20} y^{13} dxdy,$$

where $D_a = \{x^2 + y^2 \leq a^2\}$.

5. Compute the volume of the domain $U \subset \mathbb{R}^3$ defined by

$$U = \{x \geq 0, y \geq 0, 1 \leq xy \leq 2, x \leq y \leq 2x, 0 \leq z \leq x + y\}.$$

Each problem is 10 points.