Math 52H: Homework N5

Due to Friday, February 15

1. Use Fubini's theorem to compute the following multiple integrals:

a) $\int_D (x-y)dA$, where D is a triangle with vertices $(0,0), (1,0)$, and $(2,1)$.

In the following problems B is the domain in $\mathbb{R}^3$ defined by

$$B = \{0 \le x \le 1, 0 \le y \le 1, 0 \le z \le xy\}.$$

b) $\int_B x dV$,

c) $\int_B y dV$,

d) $\int_B z dV$

e) $\int_B xy dV$.

2. Find an example of a Riemann measurable subset $A \subset \{(x,y);\ 0 \le x, y \le 1\} \subset \mathbb{R}^2$ such that for infinitely many values $a \in [0,1]$ the set $A_a = \{x = a\} \cap A$ is not Riemann measurable in the line $\{x = a\}$.

3. Use change of variables formula (together with Fubini's theorem) to compute the following integrals.

a) $\int\limits_S (x^2+y^2)dV$, where $S=\{x^2+y^2\le 2x\}$.

b) $\int\limits_S \sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}dV$, where $S=\{\frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1\}$.

4. Find the area of a curvilinear quadrangle bounded by the arcs of the parabolas

$$x^2=ay, x^2=by, y^2=\alpha x, y^2=\beta x, \quad \text{where} \quad o<a<b,\ 0<\alpha<\beta.$$

Hint: introduce new variables (u,v) such that $x^2=uy, y^2=vx$.

5. In what ratio does the hyperboloid $\{x^2+y^2-z^2=a^2\}$ divides the volume of the ball $\{x^2+y^2+z^2\le 3a^2\}$.

6. Solve Exercise 8.22.2 from the online text:

Prove that if $A\subset\mathbb{R}^n$ is nowhere dense (i.e. it is closed and $\mathrm{Int}A=\varnothing$) then either $\mathrm{Vol}A=0$, or A is not measurable in the sense of Riemann. Find an example of a non-measurable nowhere dense set.

Each subproblem of 1 is 5 points. All other problems and subproblems are 10 points each.