Math 52H: List of Topics

1. Multilinear algebra

   (a) Linear and multilinear functions

   (b) Tensor product

   (c) Symmetric and skew-symmetric tensors

   (d) Exterior product and its properties

   (e) Orientation

   (f) Dualities

2. Vector Fields and Differential forms.

   (a) Differential and gradient of a smooth function.

   (b) Gradient vector field

   (c) Vector fields and differential equations

   (d) Vector fields and first order partial differential operators

   (e) Differential forms

   (f) Operator f^*, exterior product and other operations on differential forms

   (g) Exterior differential, its properties and computation

   (h) Operators on vector fields: $grad, curl$ and div

3. Integration

    (a) Riemann integral for functions and differential 1-forms

    (b) Integration of 1-forms along curves

    (c) Integration of functions over domains in $\mathbb{R}^n$

    (d) Fubini's theorem

    (e) Integration of n-forms over domains in $\mathbb{R}^n$

    (f) Change of variables formula

    (g) Computation and applications of multiple integrals

4. Manifolds and submanifolds

    (a) Construction and examples of manifolds. Manifolds with boundary

    (b) Tangent space and tangent bundles

    (c) Vector bundles and their homomorphisms.

    (d) Integration of k-forms over k-dimensional submanifolds

    (e) Orientation and co-orientation

5. Stokes's theorem.

    (a) Technical tools: partition of unity and smoothing

    (b) Stokes's formula and its special cases

    (c) Proof of Stokes's theorem

    (d) Vector analysis. Computations in curvilinear coordinates.

    (e) Integral formulas of vector analysis. Work and Flux

6. Applications of Stokes's formula

    (a) Homotopy. k-connected domains

(b) Closed and exact forms

(c) Complex-valued 1-forms, holomorphic functions and Cauchy integral formula