Math 52H: Homework N5

Due to Friday, February 17

1. Use complex numbers to find an expression of

$$\frac{\sin nx}{\sin x} \quad \text{through} \quad \cos x.$$

2. Solve Exercise 5.1 in the online text:

   (a) Compute

$$\sum_0^n \cos k\theta \;\; \text{and} \quad \sum_1^n \sin k\theta.$$

   (b) Compute

$$1 + \binom{n}{4} + \binom{n}{8} + \binom{n}{12} + \dots.$$

3. Compute

   a)

$$\int_\gamma \frac{xdx + ydy + zdz}{(x^2 + y^2 + z^2)^{\frac{3}{2}}},$$

   where the path $\gamma : [0,1] \to \mathbb{R}^3$ is given by

$$\gamma(t) = (t^3 + 1, \sin\frac{\pi t}{2} + 1, 2^t).$$

b)

$$\int\limits_\gamma \frac{xdy - ydx}{x^2+y^2},$$

where $\gamma : [0,\pi] \to \mathbb{R}^2$ is given by

$$\gamma(t) = \left(1 + \frac{1}{2}\sin\frac{t^2}{\pi}, \cos\frac{t^3}{2\pi^2}\right).$$

4. Consider in $\mathbb{R}^3$ a differential 1-form

$$\alpha = (x^2 - x + y + z - yz)dx + (y^2 + x - y + z - xz)dy + (z^2 + x + y - z - xy)dz.$$

a) Prove that α is exact and find its primitive, i.e. the function F such that $dF = \alpha$.

b) Compute

$$\int_\gamma \alpha,$$

where the path $\gamma : [0,\pi] \to \mathbb{R}^3$ is defined by the formula

$$\gamma(t) = (2^{\sin t}, \cos^2 t + 1, \sin t + 2).$$

Problem and each subproblem of 2, 3 and 4 is 10 points.