

# Math 52H: Homework N7

Due to Friday, February 25

1. Use Fubini theorem to compute the following multiple integrals:

a)  $\int_D (x - y) dA$ , where  $D$  is a triangle with vertices  $(0, 0)$ ,  $(1, 0)$ , and  $(2, 1)$ .

In the following problems  $B$  is the domain in  $\mathbb{R}^3$  defined by

$$B = \{0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq xy\}.$$

b)  $\int_B x dV$ ,

c)  $\int_B y dV$ ,

d)  $\int_B z dV$

e)  $\int_B xy dV$ .

2. Use change of variables formula (together with Fubini theorem) to compute the following integrals.

a)  $\int_S (x^2 + y^2) dV$ , where  $S = \{x^2 + y^2 \leq 2x\}$ .

b)  $\int_S \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}} dV$ , where  $S = \{\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1\}$ .

3. Find the area of a curvilinear quadrangle bounded by the arcs of the parabolas

$$x^2 = ay, x^2 = by, y^2 = \alpha x, y^2 = \beta x, \quad \text{where } 0 < a < b, 0 < \alpha < \beta.$$

Hint: introduce new variables  $(u, v)$  such that  $x^2 = uy, y^2 = vx$ .

4. In what ratio does the hyperboloid  $\{x^2 + y^2 - z^2 = a^2\}$  divide the volume of the ball  $\{x^2 + y^2 + z^2 \leq 3a^2\}$ .

Problem Each subproblem of 1 is 5 points. All other problems and subproblems are 10 points each.