

Math 52H: Homework N6

Due to Friday, February 18

1. The Cantor set $C \subset [0, 1]$ is defined as follows. First remove from $[0, 1]$ the open interval $(\frac{1}{3}, \frac{2}{3})$. Then divide each of the remaining 2 intervals in 3 equal parts, and remove central open sub-intervals. Continue this process, removing at each step the central open third of each of the remaining intervals. What is left as the result of this infinite process is the Cantor set C . Prove that $\text{Vol}_1 C = 0$.

2. Compute

a) $\int_{\gamma} \frac{xdx+ydy+zdz}{(x^2+y^2+z^2)^{\frac{3}{2}}}$, where the path $\gamma : [0, 1] \rightarrow \mathbb{R}^3$ is given by

$$\gamma(t) = (t^3 + 1, \sin \pi t + 2, 2^t).$$

b) $\int_{\gamma} \frac{xdy-ydx}{x^2+y^2}$, where $\gamma : [0, \pi] \rightarrow \mathbb{R}^2$ is given by

$$\gamma(t) = \left(1 + \frac{1}{2} \sin \frac{t^2}{\pi}, \cos \frac{t^3}{2\pi^2} \right).$$

3. Consider in \mathbb{R}^3 a differential 1-form

$$\alpha = (x^2 - x + y + z - yz)dx + (y^2 + x - y + z - xz)dy + (z^2 + x + y - z - xy)dz.$$

- a) Prove that α is exact and find its primitive, i.e. the function F such that $dF = \alpha$.
- b) Compute $\int_{\gamma} \alpha$, where the path $\gamma : [0, \pi] \rightarrow \mathbb{R}^3$ is defined by the formula

$$\gamma(t) = (2^{\sin t}, \cos^2 t + 1, \sin t + 2).$$

Problem 1 and each subproblem of 2 and 3 is 10 points.