Math 52H: List of Topics

1. Multilinear algebra
   (a) Linear and multilinear functions.
   (b) Tensor product.
   (c) Symmetric and skew-symmetric tensors.
   (d) Exterior product and its properties.
   (e) Orientation.
   (f) Hodge duality.
2. Vector Fields and Differential forms.
   (a) Differential and gradient of a smooth function.
   (b) Vector fields and differential equations.
   (c) Gradient vector field.
   (d) Differential forms.
   (e) Operator f^*, exterior product and other operations on differential forms.
   (f) Exterior differential, its properties and computation.
   (g) Operators on vector fields: $grad, curl$ and div
3. Volume and Multiple integrals

(a) Riemann integrable functions and Riemann integral.

(b) Sets of volume 0. Definition of volume of a bounded subset of $\mathbb{R}^n$.

(c) Iterated integrals and Fubini's theorem.

(d) Change of variables formula.

(e) Computation and applications of multiple integrals.

4. Integration of differential forms.

(a) Integration of k-forms over k-space.

(b) Curves, surfaces and submanifolds.

(c) Integration of k-forms over k-dimensional submanifolds.

5. Stokes's theorem.

(a) Orientation and co-orientation.

(b) Proof of Stokes's theorem.

(c) Relation between integration of forms and functions.

(d) Vector analysis. Computations in curvilinear coordinates.

(e) Integral formulas of vector analysis.

(f) Applications of Stokes's formula in Physics and Topology.