15. To estimate $(2.001)^5$, we'll find the linearization of $f(x) = x^5$ at $a = 2$. Since $f'(x) = 5x^4$, $f(2) = 32$, and $f'(2) = 160$, we have $L(x) = 32 + 160(x - 2) = 80x - 128$. Thus, $x^5 \approx 80x - 128$ when $x$ is near 2, so

$(2.001)^5 \approx 80(2.001) - 128 = 160.08 - 128 = 32.08$.

16. To estimate $e^{-0.015}$, we'll find the linearization of $f(x) = e^x$ at $a = 0$. Since $f'(x) = e^x$, $f(0) = 1$, and $f'(0) = 1$, we have $L(x) = 1 + 1(x - 0) = x + 1$. Thus, $e^x \approx x + 1$ when $x$ is near 0, so $e^{-0.015} \approx -0.015 + 1 = 0.985$.

25. (a) $y = e^{x/10} \Rightarrow \frac{dy}{dx} = \frac{1}{10}e^{x/10}$

(b) $x = 0$ and $dx = 0.1 \Rightarrow \frac{dy}{dx} = \frac{1}{10}e^{0/10}(0.1) = 0.01$.

$\Delta y = f(x + \Delta x) - f(x) = e^{(x+\Delta x)/10} - e^x = e^{(0.01)} - e^0 = e^{0.01} - 1 \approx 0.00101$

28. (a) $A = \pi r^2 \Rightarrow dA = 2\pi r dr$. When $r = 24$ and $dr = 0.2$, $dA = 2\pi(24)(0.2) = 9.6\pi$, so the maximum possible error in the calculated area of the disk is about $9.6\pi \approx 30$ cm².

(b) Relative error = $\frac{\Delta A}{A} \approx \frac{dA}{A} = \frac{2\pi r dr}{\pi r^2} = \frac{2 dr}{r} = \frac{2(0.2)}{24} = \frac{0.4}{24} = \frac{1}{60} = 0.016$.

Percentage error = relative error $\times 100\% = 0.016 \times 100\% = 1.6\%$.

30. For a hemispherical dome, $V = \frac{2}{3}\pi r^3 \Rightarrow dV = 2\pi r^2 dr$. When $r = \frac{1}{2}(50) = 25$ m and $dr = 0.05$ cm = 0.0005 m,

$dV = 2\pi(25)^2(0.0005) = \frac{25\pi}{8}$, so the amount of paint needed is about $\frac{25\pi}{8} \approx 2$ m³.

§ 4.5

5. This limit has the form $\frac{0}{0}$. We can simply factor the numerator to evaluate this limit.

$$\lim_{x \to 1} \frac{x^2 - 1}{x + 1} = \lim_{x \to 1} \frac{(x + 1)(x - 1)}{x + 1} = \lim_{x \to 1} (x - 1) = -2$$

7. This limit has the form $\frac{0}{0}$.

$$\lim_{x \to (\pi/2)^+} \frac{\cos x}{x - (\pi/2)^+} = \lim_{x \to (\pi/2)^+} \frac{-\sin x}{1 - \sin x} = \lim_{x \to (\pi/2)^+} \frac{-\sin x}{-\cos x} = \lim_{x \to (\pi/2)^+} \tan x = -\infty.$$
\[ \lim_{x \to \infty} \frac{1 - e^{-2x}}{\sec x} = \frac{1 - 1}{1} = 0. \text{ L'Hospital's Rule does not apply.} \]

29. This limit has the form $\infty \cdot 0$. \[ \lim_{x \to \infty} x^3 e^{-x^2} = \lim_{x \to \infty} \frac{x^3}{e^{x^2}} = \lim_{x \to \infty} \frac{3x^2}{2xe^{x^2}} = \lim_{x \to \infty} \frac{3x}{2e^{x^2}} = \lim_{x \to \infty} \frac{3}{4x e^{x^2}} = 0 \]

\[ 4. f(x) = 2x + 3x^{1.7} \implies F(x) = x^2 + \frac{3}{2}x^{2.7} + C = x^2 + 3x^{0.7} + C \]

5. \[ f(x) = 3\sqrt{2}x + \frac{5}{x^6} = 3x^{1/2} + 5x^{-6} \text{ has domain } (-\infty, 0) \cup (0, \infty), \text{ so} \]

\[ F(x) = \begin{cases} \frac{3}{2}x^{1/2+1} + 5 \frac{x^{-6+1}}{x^6+1} + C_1 = 2x^{3/2} - x^{-5} + C_1 & \text{if } x < 0 \\ \frac{3}{2}x^{3/2} - x^{-5} + C_2 & \text{if } x > 0 \end{cases} \]

12. \[ f(x) = \frac{x^2 + x + 1}{x} = x + 1 + \frac{1}{x} \implies F(x) = \begin{cases} \frac{1}{2}x^2 + x + \ln|x| + C_1 & \text{if } x < 0 \\ \frac{1}{2}x^2 + x + \ln|x| + C_2 & \text{if } x > 0 \end{cases} \]

22. \[ f'(x) = 4\sqrt{1-x^2} \implies f(x) = 4\sin^{-1}x + C. \quad f\left(\frac{1}{2}\right) = 4\sin^{-1}\left(\frac{1}{2}\right) + C = 4 \cdot \frac{\pi}{6} + C \text{ and } f\left(\frac{1}{2}\right) = 1 \implies \frac{2\pi}{3} + C = 1 \implies C = 1 - \frac{2\pi}{3}, \text{ so } f(x) = 4\sin^{-1}x + 1 - \frac{2\pi}{3}. \]

23. \[ f''(x) = 24x^2 + 2x + 10 \implies f'(x) = 8x^3 + x^2 + 10x + C. \quad f'(1) = 8 + 1 + 10 + C \text{ and } f'(1) = -3 \implies 19 + C = -3 \implies C = -22, \text{ so } f'(x) = 8x^3 + x^2 + 10x - 22 \text{ and hence, } f(x) = 2x^4 + \frac{1}{2}x^3 + 5x^2 - 22x + D. \quad f(1) = 2 + \frac{1}{2} + 5 - 22 + D \quad \text{and} \quad f(1) = 5 \implies D = 22 - \frac{7}{2} = \frac{53}{2}, \text{ so } f(x) = 2x^4 + \frac{1}{2}x^3 + 5x^2 - 22x + \frac{53}{2}. \]

24. \[ f'''(x) = 4 - 6x - 40x^3 \implies f''(x) = 4x - 3x^2 - 10x^4 + C. \quad f''(0) = C \text{ and } f''(0) = 1 \implies C = 1, \text{ so} \]

\[ f'(x) = 4x - 3x^2 - 10x^4 + 1 + \text{and hence, } f(x) = 2x^2 - x^3 - 2x^5 + x + D. \quad f(0) = D \quad \text{and} \quad f(0) = 2 \implies D = 2, \text{ so} \]

\[ f(x) = 2x^2 - x^3 - 2x^5 + x + 2. \]

31. \[ f(0) = -1 \implies 2(0) + C = -1 \implies C = -1. \text{ Starting at the point } (0, -1) \text{ and moving to the right on a line with slope 2 gets us to the point } (1, 1). \text{ The slope for } 1 < x < 2 \text{ is 1, so we get to the point } (2, 2). \text{ Here we have used the fact that } f \text{ is continuous. We can include the point } x = 1 \text{ on either the first or the second part of } f. \text{ The line connecting } (1, 1) \text{ to } (2, 2) \text{ is } y = x, \text{ so } D = 0. \text{ The slope for } 2 < x \leq 3 \text{ is } -1, \text{ so we get to } (3, 1). \text{ } f(3) = 1 \implies -3 + E = 1 \implies E = 4. \text{ Thus,} \]

\[ f(x) = \begin{cases} 2x^2 + C & \text{if } 0 \leq x < 1 \\ x + D & \text{if } 1 < x < 2 \\ -x + E & \text{if } 2 < x \leq 3 \end{cases} \]

Note that \( f'(x) \) does not exist at \( x = 1 \) or at \( x = 2 \).