1. Show that the viscous Burgers equation \( u_t + uu_x = \epsilon u_{xx} \) has a traveling-wave solution of the form \( u^\epsilon(x,t) = w(x-\epsilon t) \) by deriving an ODE for \( w \) and verifying that this ODE has solutions of the form

\[
    w(\xi) = u_r + \frac{1}{2}(u_l - u_r) \left[ 1 - \tanh \left( \frac{(u_l - u_r)\xi}{4\epsilon} \right) \right],
\]

when \( u_l > u_r \) with the propagation speed \( s \) given by the Rankine-Hugoniot condition. Note that \( w(\xi) \to u_l \) as \( \xi \to -\infty \) and Note that \( w(\xi) \to u_r \) as \( \xi \to \infty \). Sketch this solution and indicate how it varies as \( \epsilon \to 0 \). What happens to this solution if \( u_l < u_r \) and why is there no traveling-wave solution with limiting values of this form?

2. Solve the scalar equation \( u_t + f(u)_x = 0 \) with 2\( \pi \)-periodic boundary conditions, using the Roe scheme and the Lax-Friedrichs scheme. The flux \( f(u) \) is given as

\[
\begin{align*}
  (a) & \quad f(u) = u^2/2 \quad \text{(Burgers)} \\
  (b) & \quad f(u) = u^2/(u^2 + (1-u)^2) \quad \text{(Buckley-Levert)}
\end{align*}
\]

The initial condition is \( u_0(x) = \sin(x) + 2 \). Plot the results at time \( T = 1 \) with \( N = 20, 40, 80, 160 \) cells. Compute the \( L^1 \) and \( L^\infty \) errors for fluxes (a) and (b) at time \( T = 1 \).

3. Show that the Engquist-Osher scheme

\[
    \bar{w}_j^{n+1} = \bar{w}_j^n - \frac{\lambda}{2} \left[ f(\bar{w}_{j+1}^n) - f(\bar{w}_{j-1}^n) \right] + \frac{\lambda}{2} \left[ \int_{\bar{w}_{j+1}^n}^{\bar{w}_{j+1}^n} |a(\xi)|d\xi - \int_{\bar{w}_{j-1}^n}^{\bar{w}_{j-1}^n} |a(\xi)|d\xi \right],
\]

is equivalent to approximating the local Riemann problems on the cell-interfaces with rarefaction waves (for a convex \( f(u) \)).