Math 222A - HW 1

Doron Levy

Due February 12

1. Write the Lax-Wendroff scheme for the heat equation

\[ u_t = k u_{xx}, \quad k = \text{const}. \]

2. What is the order of accuracy of the following Leap-Frog-type scheme,

\[ v(x, t + k) = v(x, t - k) + 2ka \left[ \frac{4D_0(h) - D_0(2h)}{3} \right] v(x, t), \]

as an approximation for the wave equation, \( u_t = au_x \). Here

\[ D_0(h)v(x, t) = \frac{v(x + h, t) - v(x - h, t)}{2h}. \]

3. Write a computer program for solving the periodic, scalar problem

\[ u_t = 4u_x, \quad 0 \leq x \leq 2\pi, \]

subject to the initial data \( u_0(x) = 1.5 - 2.5 \cos x \), using the following schemes: Lax-Friedrichs and Lax-Wendroff.

Use the following parameters:

\[ N = 10, 20, 40, 80, \quad \text{CFL} = 0.9, \quad T = 2. \]

Estimate the order of accuracy of these methods in the \( L^2 \) and the \( L^\infty \) norms.

4. Consider

\[ u_t = 2u_x, \quad -\pi \leq x \leq \pi, \]  \hspace{1cm} (1)

with periodic boundary conditions and initial data \( u(x, 0) = f(x) \).

Approximate the solution of (1) for \( f(x) = \sin(3x) \) with a pseudo-spectral method. Compute the solution at time \( T = 1 \) using \( N \) modes, where \( N = 8, 16, 32, 64 \). Compute the error and estimate the order of convergence.