Part One:

Warm-up

Problem 1 For each of the following pairs of numbers, compute the GCF and the LCM:

a) 4 and 6;
b) 10 and 21;
c) 20 and 24;
d) 10 and 34.

Problems

Problem 2 Do Activity 8F on page 162 of Beckmann.

Problem 3 For each of the pairs of numbers in Problem 1, write down:

a) the prime factorization of each of the numbers given in exponential form;
b) the prime factorization of the GCF in exponential form;
c) the prime factorization of the LCM in exponential form.

After you have done this for each of the pairs, come up with a conjecture relating the prime factorizations of the numbers given to the prime factorizations of the GCF and the LCM. Using your conjecture, write out a procedure that you could use to find the GCF and the LCM of two numbers if you were given their prime factorizations.

Problem 4

a) It is a fact that there are integers a and b such that $2 = 4a + 6b$. Find a and b.
b) Can you find integers a and b such that $1 = 4a + 6b$?
c) Can you find integers a and b such that $3 = 4a + 6b$?

Part Two:

Warm-up

Problem 1 For each of the following pairs of numbers, compute the GCF and the LCM *using the Euclidean algorithm*:

a) 4 and 6;
b) 10 and 21;
c) 20 and 24;
d) 10 and 34.

Problems

Problem 2 Finish Problem 4 from Part One of this worksheet if you haven't already.

Problem 3 It is a fact that for any whole numbers m and n, we can always find integers a and b such that

$$\mathrm{GCF}(m, n) = a \cdot m + b \cdot n$$

For example, the GCF of 3 and 5 is 1 and we can write $1 = (-3) \cdot 3 + 2 \cdot 5$.
For each of the pairs of numbers in Problem 1, write the GCF as $a \cdot m + b \cdot n$. Hint: At each line, solve for the remainder, and back substitute from the last line up.

Looking Forward

Problem 4 Do Activity 8J on page 166 of Beckmann.