

CM Seminar

Goal: Come to as complete an understanding of the theory of Complex Multiplication and its applications as time allows.

Topics: The topics to be covered, listed in order and with the associated references, are the following.

- CM for elliptic curves: ray and ring class fields of imaginary quadratic fields, reciprocity laws, L-functions of elliptic curves with CM. [Si], [L1], [M5]
- Review of abelian varieties: Hodge theory, polarizations, endomorphism rings, CM abelian varieties [L2], [M1], [Mu], [SeTa]
- The theorem of Shimura-Taniyama: statement and proof [L2], [Sh], [M2], [M3]
- Applications to zeta-functions of abelian varieties with CM [SeTa], [M2], [M4]
- Canonical models again? [M3]
- Tate-Honda theory? And applications? [Ta]
- The generalizations of Tate and Deligne: statements [M2]
- Applications to zeta-functions of abelian varieties with CM [M2]
- Applications to Hilbert's Twelfth Problem [M2]

References. These will undoubtedly lead to others.

- [L1] S. Lang. *Introduction to modular forms*. Grundlehren der mathematischen Wissenschaften, No. 222. Springer-Verlag, Berlin-New York, 1976.
- [L2] — *Complex multiplication*. Grundlehren der Mathematischen Wissenschaften, No. 255. Springer-Verlag, New York, 1983.
- [M1] J. Milne. Abelian varieties. *Arithmetic geometry (Storrs, Conn., 1984)*, 103–150, Springer, New York, 1986.
- [M2] — Abelian Varieties with Complex Multiplication (for Pedestrians) . www.jmilne.org
- [M3] — Introduction to Shimura varieties. www.jmilne.org
- [M4] — On the arithmetic of abelian varieties. *Invent. Math.* 17 (1972), 177–190.
- [M5] — Modular functions and modular forms. www.jmilne.org
- [Mu] D. Mumford. *Abelian Varieties*
- [SeTa] J-P. Serre and J. Tate. Good reduction of abelian varieties. *Ann. of Math.* (2) 88 1968 492–517.
- [Sh] G. Shimura *Abelian varieties with complex multiplication and modular functions*. Princeton Mathematical Series, 46. Princeton University Press, Princeton, NJ, 1998.
- [Si] J. Silverman, *Advanced Topics in the Theory of Elliptic Curves*. Springer, 1996.
- [Ta] J. Tate. Classes d'isogénie des variétés abéliennes sur un corps fini (d'après T. Honda). *Seminaire Bourbaki*. Vol. 1968/69: Exposé 347–363. *Lecture Notes in Mathematics*, Vol. 179

